DESIGNING A DIDACTICAL SITUATION ON SYMBOL SENSE OF MINUS SIGN IN LEARNING ARITHMETIC OPERATION OF INTEGER

NYIAYU FAHRIZA FUADIAH

Doctoral Student of Math Education,
Universitas Pendidikan Indonesia.
Email: n_fahriza@yahoo.co.id

Abstract

Mathematics textbooks are usually a reference for teachers to convey the material in class. The existence of textbooks help teachers to facilitate the learning process because their contain some descriptions of concepts, example problems, and evaluations so that the concepts and flow of thought contained in the textbook can guide teachers in the learning process. The study on mathematics textbooks of primary and junior high school on the material negative numbers less meaning the symbol of minus sign (*symbol sense of minus sign*) beforehand as the initial concept of negative numbers so that students do not understand the meaning of negative numbers at the next level. The learning process is more likely to emphasize the procedure only that resulting in a leap of information and thinking experienced by students. Finally, character of student's thinking forced to jump from the concrete into abstract thinking. Based on the results of a study conducted by researcher on the students of seventh grade, the concept of negative numbers became an obstacle in performing arithmetic operations of numbers. In integers which there are negative integers in shows that most of them do not understand the meaning of the symbol minus sign as a prerequisite to understanding arithmetic operations involving negative integers. Therefore, teachers need to design a learning through a didactical situation on symbol sense of minus sign which appropriate to the character of the student's thinking.

Keywords: *negative numbers, symbol sense, minus sign*

1. Intoduction

As children, we first encounter plus and minus signs as symbols of addition and subtraction of counting numbers and, perhaps, of fractions (Galbraith, 1974). Addition even been introduced to children from an early age. Subraction then submitted as advanced concept of addition. When it was first introduced, it might be easy for the child to determine 'what two cakes should be added in order to become 5 cakes?' when compared with 'how many 5 cakes are substracted three cakes?' This question becomes abstract for children when asked 'what does 5-3? Subtraction, on the other hand, is far more complex for children. Then, they recognized a symbol of minus sign beside plus sign. For them the minus sign means a subtraction operation with the meaning 'take away'. This concept then kept embedded in students until then recognized the concept of negative numbers.

We often associate the terms integers and negative numbers with difficulty, frustration, and confusion so many of us probably still remember our initial struggles to understand negative numbers and how to operate with them (Bishop et.al, 2011). Bofferding (2014) stated that many students struggle to make sense of negative integer concepts because they seemingly conflict with their established understanding of nonnegative numbers. As revealed by Bofferding, students start learning whole number concepts in kindergarten (or earlier), fractions in second or third grade, and decimals in fourth grade but, typically, do not learn negative number concepts until sixth and seventh grade. The *larger minus smaller* rule contributes to students solving problems such as $24 - 19$ as $29 - 14$ because they switch the digits in the ones place (Fuson in Bofferding, 2014). Even without hearing this rule, many students incorrectly solve problems such as $3 - 5$ as $5 – 3$.

Before they receive instruction on negative numbers, students primarily rely on the binary meaning of the minus sign, interpreting all minus signs as subtraction signs (Bofferding, 2010); this interpretation may be one reason why students call the negative sign a “dash” or “minus.” However, students need to understand that the “dash,” or negative sign, serves to designate numbers that are ordered before zero and have different values than positive numbers. In Indonesia, negative numbers are encountered in mathematics curriculum at the beginning of secondary school, ie at the seventh grade (Kemdikbud, 2013). In fact, these materials have been given to fourth grade students in elementary school. Unlike positive number, negative number has not perception referential which is clear, and therefore, students should try harder to learn about negative number (Blair. et al, 2012). The concept of subtraction as the meaning of 'take away' are not applicable to students when faced with, for example, 2 – (-5). Based on auhtor's observation, there was found confuse in meaning 'take away -5' and when –(-5) changed into +5 on the students. This may not be the case if they were introduced to symbol of minus sign as 'opposite of'so they know that –(-5) as oppsosite of -5 meaning therefore its was obtanined +5.

Mathematics textbooks did not anticipate the obstacles faced by students in understanding the abstract concept of negative numbers. Based on author's, the 7th grade mathematics textbooks mostly prefer to procedure with memorization the mathematics formula that unfamiliar to them (see Fuadiah, 2015). In part the books are less attention to flow of thiking on students who are in the informal to the formal transition thinking. At this stage, students still use thinking arithmetic than algebra. A study conducted by the author, many students stating that -4 + 5 as -9. This is due to their latest information that the negative numbers with the positive numbers it will be negative numbers. Therefore, the textbook should be able to accommodate the students's condition to use the concepts of the symbol of minus sign thay they had previously understood so there is no 'thinking leap' on them.

The existence of textbooks would not be separated from the lesson plan prepared by the teacher. The main function of the learning plan is to give students learning opportunity so that a teacher must plan what would probably happen during the learning process (Sanchez & Valcarcel, 1999). Brousseau (2002) affirmed that the role of the teacher is to encourage mathematical ideas in a context through the inquiry process. Therefore, we need to realize entirely that it is important for teachers to design learning with didactic design to anticipate all possible student responses on a didactic situation (Suryadi, 2013). The application of the theory of didactic situations through the design of a didactic situation created by the teacher in the learning activities in the classroom are expected to develop the potential of students, which they can construct their own knowledge that will be achieved through a series of processes of abstraction. Action and feedback through a strategy will allow the establishment of a new knowledge.

2. Theoretical Background

Theory of Didactical Situation

The theoretical idea that stays behind this approach is the main role given to the relation between students learning process and the environment where the learning happens (Manno, 2006). The first step in this theoretical approach is the analysis of teaching-learning phenomenon within the triangle teacher-knowledge-student (Didactic Triangle). Further, Manno also explained that the double implication of this triangle suggests that we are facing a complex interaction that works back and forward. When analysing the triangle it is important that no one of the members takes a main role, every study of the topic teaching-learning has to consider the three members at the same level.

Theory of Didactical Situation (TDS) is interested in didactical situations , that is, those designed and utilized with teaching and learning aims. Brousseau distinguishes two possible perspectives on didactical situations: a vision of these as the student's environment organized and piloted by the teacher; and a broader vision including the teacher and the educational system itself (Artigue et. al, 2014).

Reffering to Brousseau, Artigue et.al (2014, pp. 49-50) explained that there are some charateristics of TDS. The first important characteristic of TDS is the attention it pays to mathematics and its epistemology. In the theory, this sensibility is expressed in different ways, notably through the reference to Bachelard's epistemology and the didactic conversion of his notion of *epistemological obstacle*, and also through the notion of *fundamental situation*. Referring to Bachelard's studies in physics which led to a list of obstacles of epistemological nature, Brousseau (2002, p.83) extends its application to the fi eld of didactics of mathematics, defining epistemological obstacles as forms of knowledge that have been relevant and successful in particular contexts, including often school contexts, but that at some moment become false or simply inadequate, and whose traces can be found in the historical development of the domain itself.

A second important distinction in TDS is linked to the following epistemological characteristic: mathematical knowledge is something that allows us to act on our environment, but the pragmatic power of mathematics is highly dependent on the specific language it creates, and on its forms of validation. This characteristic reflects in TDS through the distinction between three particular types of situations: *situations of action* , *situations of formulation* , and *situations of validation* (Brousseau, 2002; Kislenko, 2005; Perrin-Glorian, 2005; Manno, 2006; Wisdom. 2014).

The third important characteristic refers to students' cognitive dimension, particularly to the combination of the two processes *adaptation* and *acculturation*. Regarding adaptation, Brousseau's discourse shows an evident proximity with Piagetian epistemology:

> *the student learns by adapting herself to a milieu which generates contradictions, difficulties and disequilibria, rather as human society does. This knowledge, the result of the students' adaptation, manifests itself by new responses which provide evidence for learning. (Brousseau 2002, p.30)*

TDS key constructs take that teaching is an activity needing to conciliate two processes: independent adaptation and acculturation (Perrin-Glorian, 2005). Independent adaptation through the notions of *a-didactical situation* and *milieu* and acculturation through the notions of *didactical situation* and *didactical contract.*

A-didactical situation, with respect to knowledge *S*, is that situation that contains all the conditions that permit the student to establish a relationship with *S*, regardless of the teacher. The actions that the student does, and the answers and arguments that she produces depend on her relationship (no completely explicit) with *S*, i. e. with the "problem" that she must solve or wit the difficulty that she must overcome. In this case, a process of devolution of responsibility is in action (Samaniego & Barrera, 1999). ***The milieu*** is the system with which the students interact in the a-didactical situation and an essential role of the teacher or the researcher is to organize this milieu. It includes material and symbolic resources, possibly calculators, computer devices, or all types of machinery (Artigue, 2014). In *a-didactical Situations* it is the students who have the initiative and the responsibility for what comes of the Situation (Brousseau et.al, 2014, p. 147). The teacher thus delegates part of the care for justifying, channeling and correcting the students' decisions to a *milieu*.

Didactical situation, with respect to knowledge S, is that situation design explicitly to encourage S. We can consider as didactical all the tasks done in a classroom with which the teacher intents to teach *S*, and with which the student is forced to learn *S* (Samaniego & Barrera, 1999). In *didactical Situations* , the teacher maintains direct responsibility for all stages of the lesson. She tells the students her intentions, what they will have to do, and what the results should be (Brousseau et.al, 2014). ***The didactical contract*** is the only rule and strategy of the didactical situation and it is strictly related to knowledge. Often students do not answer teachers' questions on the basis of the content that teachers mean to give them, but on the basis of what they think teachers expect from them (Manno, 2006). Miyakawa and Winslow (2009) stated that didactic contract is a contract that governs the responsibility of students and teachers and their interaction in the learning process.

The relationships between these processes through the dual notions of *devolution* and *institutionalization* (Artigue et.al, 2014). Further, Artigue explained also that through *devolution* , the teacher makes her students accept the mathematical responsibility of solving the problem without trying to decode her didactical intention, and maintains it, creating thus the conditions for learning through adaptation. Through *institutionalization* , the teacher helps students to connect the contextualized knowledge they have constructed in the a-didactical situation to the target cultural and institutional knowledge.

Based on TDS, mathematics learning be implemented in three steps as revealed by Brousseau (2002, pp. 8-13) and that are described by Manno (2006);

1. Action

Students start working on the problem and produce new hypothesis and strategies proved
by new experiences. The interaction between students and the environment (other students,
the problem context, the teacher) is useful to create some first strategies and is called "dialectic of the action". At this moment students build an implicit model: a set of rules relations useful to take new decisions without being conscious of it or needing to express them in an explicit way.

2. Formulation

Now the context gives students the chance to create their own implicit model, to express strategies with words, to discuss and preserve them, making other student accept them. To do so every one will have to use a language understood by other students. The communication exchange between students lead them to a keep going strategy creation, we are in the dialect of the formulation.

3. Validation

Models that come from the previous steps can be accepted or refused by other students. In
their group all students have an equal grade so they can discuss their strategies, the hypothesis they all agree on becomes a theorem. Students often accept wrong

theories, the a-didactical situation lead them to a review of their process to make sure that they use a proper strategy. In this way mistakes are a basic point in the knowledge building process. With the validation step it is possible to give the mathematical concept a shape that in the traditional way of teaching is a starting and never a ending moment.

Symbol Sense of Minus Sign

The minus sign is used in three common ways. The three problems in table 1 use the symbol "–" that we refer to, in general, as the minus sign. However, each problem may elicit a different meaning for students. The first meaning is shown in problem 1, in which the minus sign indicates subtraction, the original use of the symbol that young children en-counter. In problem 2, the minus sign is part of the symbolic representation for a negative number, in this case, "negative 2." In problem 3, however, the first minus sign may be viewed as the *opposite of* so that one could read - -4 as "the opposite of negative 4" rather than students' more common reading of "negative negative 4" (see Bofferding 2014; Lamb, et al, 2012; Vlassis 2008).

Table 1. Three meanings of the minus sign

Problem	Meaning of the Minus Sign
1. 5 – 8 =	Subtraction as a binary operation
2. + 5 = -2	A symbolic representation for a negative number
3. Which is larger, - -4 or -4?	The *opposite of*, a unary operation

In problem 1, the minus sign func-tions as a *binary operator* in that two inputs are used to produce one output. Addition, subtraction, multiplication, and division are examples of binary operators. For example, subtraction is a binary operator because the inputs of 5 and 8 result in one output, -3. In problem 3, in contrast, the minus sign is used as a *unary operator* in that it involves only one input and one output. When one thinks of –(-4) as "the opposite of negative 4," then one is view-ing the first minus sign as the unary operator, the *opposite of*. However, in problem 2, some people may view the minus sign in -2 as a unary operator, not as part of the number but instead as the *opposite of* 2. One who can view -2 in both ways can be said to flexibly hold both meanings of the minus sign. Although students may initially face difficulties because three meanings are assigned to the same symbol, having the same symbol represent several ideas is important.

On the seventh grade students, based on author's observation, negative integers become separate obstacles in arithmetic operation of integer. This is in line with Larsen (2012) that negative numbers are an abstract concept for which students need phenomenological guidance in order to avoid epistemological obstacles. The author argues that the symbol of minus sign must be understood first by the students before understanding integer arithmetic operations because their understanding of these symbols of minus sign will be useful when they perform integer arithmetic operations. For example, in operation 8 – (-3), through the meaning of opposite as one of the meanings of the minus sign, the student will know that –(-3) means the opposite of -5 is +5 so 8 – (-3) equal to 8 + 3. For the other example, -4 will interprete as the opposite direction from 4 or +4. It expected to be easier to understand by the students than memorized the formulas of integer arithmetic operations.

3. Result and Discussion

Based on the background and the theoretical review that has been disclosed, the author tried to design a didactic situation in learning activities that can be carried out by the teacher before entering the integer operations in seventh grade. Learning design for the *Symbol Sense of Minus Sign* is based on the theory of didactical situations through three stages of the situation by paying attention to the meaning of the minus sign. This design is hypothetical so that still need testing to see the extent of its influence on students (see table 2). In this design, the author included prediction of response or feedback of students and the anticipations can be done by the teacher.

This steudent's predictions of response certainly be evolve or change by trials. Thus the design can be revised based on the real response that appears and findings in the field.

Tabel 2. Design of Didactical Situation on *Symbol Sense of Minus Sign*

Learning Goal	Phase	Didactical Situation (teacher's input)	Student Activity	Mathematical Hyphotesis
1. Students know the meaning of the minus sign as "binary function" (subtraction) 2. Students know the meaning of the minus sign as "unary function" (number sign) 3. Students know the meaning of the minus sign as "symmetric function" (opposite of)	Action	Teacher prepares some questions in daily life associated with the minus sign on a piece of paper. The questions such as: ➢ Mother has 12 cakes. Eight cakes given to your sister. How do you calculate the remaining cakes? ➢ Rani stepped forward as much as 5 steps. How to write five steps back? ➢ What the opposite of 4?	Students determine the answers according to the questions through discussions with friend bench or their group and then paste on a piece of cardboard	Students can give some answers involving symbols of minus sign
	Formulation	Ask for the students to make a few questions or statements themselves as teachers gave earlier. The teacher asks the students attention frequent symbol. The teacher asks the students to show their work on the board.	Designing some questions and answers themselves through discussion. Pair of this questions and answers is placed on a piece of cardboard. Then they identifying the symbol of minus sign.	Students can construct the usage of minus sign
	Validation	Encourages the students to examine and discuss the answers, then provide reinforcement to the students' answers. Asks the students to determine whenever the minus sign is used, then leads to a conclusions and provides reinforcement	Students explain the answers requested to provide their own argument	Students can conclude meaning of the symbol of minus sign

One of the students' response that must be considered by the teacher is when students do not understand the meaning of questions. For example, teachers predicted there were some students do not understand if the 5 steps forward with 5 or +5, then five steps back into -5. For this, teachers need to create a new didactical contract by giving another question that is easier to understand by the students and according to the objective of the that question. The question is made in stages, for example, teachers given the initial question: "*What do you write a temperature of 50^0 C above 0?"* If the answers correctly then asked them with the question: "*How about 50^0 C below 0? Written with what?*" This step is necessary as a bridge between learning goals with the students' thinking skills. Therefore, teachers need to set up some alternatives anticipate if some obstacles present in the learning process. Accordingly, there are balance between teacher-student- material within the didactics triangle. The synergy of the three components in the didactics triangle with anticipation didactic and the pedagogical are expected creating independence of learning in students (Suryadi, 2016).

4. Conclusion and Remark

Bishop et. al (2011) recommended that although integers are not part of the first-grade curriculum, we would like teachers to be aware of ways in which they can easily enrich and extend children's mathematical thinking by building on their ideas about negative numbers when they arise naturally in the classroom. Given the success of the instructional interventions, it would be worthwhile to explore the use of similar instruction with older students, providing them with more targeted integer experiences around the multiple meanings of the minus sign and interpreting integer values from both positive and negative perspectives (Bofferding, 2014). Lamb et. al (2012) argued that these experiences will support students in developing symbol sense in relation to the minus sign that will foster their future learning as they move from middle school into high school and beyond. Students learn to interact the environment by adapting their knowledge of different strategies without the help of a teacher to a wide range of possibilities. Didactic actions of a teacher in the learning process will create a situation that can be a starting point for the process of learning.

References

Artigue, M., Haspekian, M. & Corblin-Lenfant, A. (2014). Introduction to the theory of didactical situation (TDS). In: Ahsbahs & Prediger (Eds.), *Networking of theories as a research pratice in mathematics education* (pp.47 – 65). Switzerland: Springer International Publishing.

Blair, K.P., Rosenberg-Lee, M., Tsang, J.M.., Schwartz, D.L., & Menon, V. (2012). Beyond natural numbers: negative number representation in parietal cortex. *Frontiers in Human Neuroscience*, *6*(7), 1–17. http://dx.doi.org/10.3389/fnhum.2012.0000.

Bishop, J.P., Lamb, L.L., Bishop, J.P., Philipp, R.A., Schappelle, B.P., & Whitacre, I. (2011). First graders outwit a famous mathematician. *Teaching Children Mathematics*, *17*(1), 350-358.

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders' mental models. *Journal for Research in Mathematics Education, 45*(2), 194-245. http://dx.doi.org/10.5951/jresematheduc.45.2.0194.

Brouseau, G. (2002) *Theory of Didactical Situation in Mathematics*. Dordrecht: Kluwer Academic Publishers.

Brousseau, G., Brousseau, N., & Warfield, V. (2014). *Teaching Fractions through Situations: A Fundamental Experiment.* Dordrecht: Springer.

Fuadiah, N.F. (2015). Epistemological obstacles on mathematics'learning in junior high school students: a study on the operations of integer material. In *Proceeding of International Conference On Research, Implementation And Education Of Mathematics And Sciences 2015 (ICRIEMS 2015), Yogyakarta State University, 17-19 May 2015*. Faculty of Mathematics and Sciences Yogyakarta State University.

Galbraith, M.J. (1974). Negative numbers. *International Journal of Mathematical Education in Science and Technology*, *5*(1): 83 – 90. http://dx.doi.org/10.1080/0020739740050111.

Kemdikbud. (2014). *Matematika SMP/MTs Kelas VII Semester 1*. Jakarta, Indonesia: Kementrian Pendidikan dan Kebudayaan.

Kislenko, K (2005) Student's beliefs about mathematics from the perspective of the theory of didactical situations. In C Winslow (ED.), *Didactic of mathematics-the French way* (pp. 83-96). Center For Naturfagenes Didaktis University of Copenhagen.

Lamb, L.L., Bishop, J.P., Philipp, R.A., Schappelle, B.P., Whitacre, I., & Lewis, M. (2012). Developing symbol sense for the minus sign. *Mathematics Teaching in the Middle School, 18*(1), 5-9.

Larsen, J. (2012). Epistemological obstacles of negative numbers. *Vector: The Official Journal of the BC Association of Mathematics Teachers, 53*(2), 56-60 [online]. Retrieved from https://www.researchgate.net/publication/274310485_Epistemological_Obstacles_of_Negative_Numbers.

Manno, G. (2006). Embodiment and a-didactical situation in the teaching-learning of the perpendicular straight lines concept. Doctoral Thesis. Departement of Didactic Mathematics Comenius University Bratislava.

Miyakawa, T & Winsløw, C. (2009). Didactical designs for students' proportional reasoning: an"open approach"lesson and a "fundamental situation". *Education Studies in Mathematics,72*,199–218. doi:10.1007/s10649-009-9188-y.

Perrin-Glorian, M.J. (2008). From producing optimal teaching to analysing usual classroom situations development of a fundamental concept in the theory of didactic situations: the notion of milieu. [*Online*]. Retrieved from: https://www.unige.ch/math/EnsMath/Rome2008/WG5/Papers/PERRIN.pdf

Samaniego, A., & Barrera, S. V. (1999). Brousseau in Action: Didactical Situation for Learning How To Graph Functions.[*Online*]. Retrieved from http://eric.ed.gov/?id=ED451036.

Sanchez & Varcarcel. (1999). Science teachers' views and practices in planning for teaching. *Journal of Research in Science Teaching, 36*(4): 493 – 513.

Suryadi, D. (2013). *Didactical Design Research* (DDR) dalam pengembangan pembelajaran matematika. *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika.* Bandung, Indonesia: STKIP Siliwangi.

Suryadi, D. (2016). Didactical Design Research: Upaya membangun kemandirian berpikir melalui penelitian pembelajaran. In D. Suryadi, et al (Eds): *Monograf Didactical Design Research*. Bandung, Indonesia: Rizqi Press.

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology, 21*(4), 555–570. http://dx.doi.org/10.1080/09515080802285552.

Wisdom, N.J. (2014). Meta-didactical slippages: a qualitative case study of didactical situations in a ninth grade mathematics classroom. *Dissertation*. Departement of Middle-Secondary Education and Instructional Technology Georgia State University.