ANALYSIS MOMENT INERTIA OF TIPPE TOP IN FLAT FIELD AS CONTRIBUTION TO MECHANICAL STUDIES

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Abstract

The moment of inertia of a reverse gas can be calculated by physically analyzing the parts of the return gas. The back casing consists of a cut ball and a small tube. The back of the head is a ball (sphere) which has a radius R while the handle forms a finger that has fingers. The moment of inertia back gasing is the sum of the moment of truncated inertia and the moment of inertia of the tube with the rotating axis at the center of the mass in the back gas. Backfill is an example of a rigid body system that has a holonomic style, which can move in translation and rotation.

Keywords: tippe top, holonomic force, moment of inersia

INTRODUCTION

The inertial moment of the back gas which moves in the flat plane using the Eular equation because of the unique and complex backing shape. The author is interested in analyzing the moment of inertia in the backing in the flat plane with the Eular equation. Rotational dynamics are difficult to formulate with the Eular-Lagrange equation because the dynamics of rotation contain angular velocities which are generally not derivatives of time directly from general coordinates [1]. This is because the rotation generator is not commutative, so the rotation dynamics are difficult if it is solved by the Eular-Lagrange equation. The Poincaré equation is chosen by the author because this equation can formulate the dynamics of the top back clearly. In addition, the Poincaré equation can describe a dynamic system in the form of a system of differential equations. This study is an attempt to understand backward movement by using group theory in simplifying the equation of the backing motion through the Poincaré equation. Its origin of the research on backward movement was explained in a book in 1890 by John Perry (in Cohen, 1977) who experimented with turning round stones found on the Beach. Perry explained that this round stone has a center of mass that does not coincide with the center of the stone's geometry. When the rock is rotated, the center of mass becomes higher away from the surface of the ground [2].

Explanations of the backlash movement began to be poured in several scientific articles since the 1950s, including by Pliskin (1953) which stated that the interaction of friction in the turning back to the floor plays an important role in the turning of the turning gas. While Synge in 1952 (in Pliskin, 1953) explained that the phenomenon of backward movement is a result of dynamic instability without involving friction [3]. Furthermore, Del Campo in 1955 (in Cohen, 1977) explained in detail the mathematical calculation of the role of friction in the return tops. Del Campo concluded that friction affected the reversal event in the return top [4].

METHOD

This study is a theoretical athematical study. The study was conducted by a review of some literature about the mechanical systems on tippe top case that has been previously developed and mathematical calculations.Poincaré equationcan be written by,

$$\frac{d}{dt}\left(\frac{\partial \bar{T}}{\partial s^i}\right) - c^r{}_{li}(q)s^l\frac{\partial \bar{T}}{\partial s^r} - \frac{\partial \bar{T}}{\partial \sigma^i} = S_i[5] \quad (1)$$

However, this equation requires that the discovery of quasi velocity as a direct derivative of the time of the coordinate quasi. Meanwhile, in the case of tippe top's quasi velocity is not owned directly derived from a cyclic coordinate. Therefore, Poincaré equation used in this study to analyze the dynamics of tippe top on a flat surface and the surface of the tube is Poincaré equation that is based on the reduction Routhian, which can be written as follows

$$\frac{d}{dt}\frac{\partial R}{\partial v^\rho} - \sum_{\mu=2}^{n}\sum_{\lambda=2}^{n} c^\lambda{}_{\mu\rho}v^\mu\frac{\partial R}{\partial v^\lambda} - \sum_{\mu=2}^{n} c^\lambda{}_{\mu\rho}v^\mu\beta_1 - X_\rho R = 0.[6] \quad (2)$$

In classical mechanics, the movement of rigid bodies is generally described by two analogous vector equations: $\mathbf{F} = \mathrm{d}\mathbf{p}/\mathrm{dt}$ for translation of the centre of mass, and $\mathbf{M} = \mathrm{d}\mathbf{L}/\mathrm{dt}$ for rotation around the centre of mass, with **F**the total external force, $\mathbf{p}$ the momentum, **M** the total moment of external forces, and **L** the angular momentum.

We consider the intriguing movement of the tippe top. It consists of a spherical bodyand a cylindrical stem, with the centre of mass CM displaced from the centre c of the sphere (see Fig. 1). When initially put into rotation around its axis of symmetry $\hat{\boldsymbol{e}}_\mathbf{3}$ vertical, the stemgradually moves downwards and finally the top flips over into a stable vertical rotation on thestem. Apparently the rotation has changed sign, while vector **L**has preserved its originalvertical position. Further, CM has moved upwards at the cost of a decrease in magnitude of **L.**[7]

This unexpected behaviour is explained by the action of a friction force **F**at the (slipping) contact point of the top with the surface.**F**causes a moment **M**, which can be imagined to have vector components $\mathbf{M}_{\boldsymbol{n},\boldsymbol{n}'}$ and $\mathbf{M_3}$, the latter along the axis of symmetry $\hat{\boldsymbol{e}}_\mathbf{3}$[8].

Likewise, the angular momentum **L** has components $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}'}$ and $\mathbf{L_3}$. In the beginning, $\mathbf{L_3}$ = L and $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}'}$= 0. Then, due to instability, **F**originates and the resulting $\mathbf{M_3}$ tends to decrease $\mathbf{L_3}$, while $\mathbf{M}_{\boldsymbol{n},\boldsymbol{n}'}$ starts to increase $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}'}$. As **L**remains constan, the angle $\theta$of the top's inclination will grow to fulfil proper vector addition.When $\theta = \frac{\pi}{2}$, $\mathbf{L_3}$ = 0and $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}'} = \mathbf{L}$. [9]

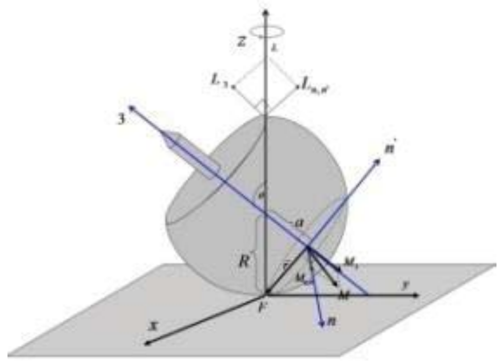

Figure 1. Tippe top inverting

Then the rotation along $\hat{\boldsymbol{e}}_3$changes sign and, again through the action of $\mathbf{M}_{n,n'}$ and $\mathbf{M}_3$, $\mathbf{L}_3$ starts to grow at the cost of $\mathbf{L}_{n,n'}$. Finally, the stem will scrape the surface (see Fig. 1) and through the action of a new frictional force $\mathbf{F}'$ with moment $\mathbf{M}'$ the top will lift itself up and strive towards a stable, though extinguishing, rotation on the stem. In fact, the component $\mathbf{L}_{n,n'}$ is extinguished by the new $\mathbf{M}_{n,n'}$ and $\mathbf{L}_3$ finally becomes equal to $\mathbf{L}$[10].

RESULT AND DISCUSSION

Moment of Inertia Tippe Top

Backshift is a sphere with radius R which has a symmetry mass distribution but not spherical symmetry, so the center of mass and center of geometry do not coincide. The line that connects the center of mass and the center of geometry is the axis of inertial symmetry of this line which is perpendicular to the axis of the moment of inertia tensor in the ball which has the same two main inertial moments namely $I_n = I_{n'} = I$, while the moment of inertia along the symmetry axis is denoted by I3, so large I and I3 can be specified [11]. The first thing to do is to find the main moment in the turning top at the center of mass as shown in Figure 1. The head of the return gas is the truncated part of the sphere which has a radius R while the handle is tubular which has radius b and its cone shaped radius rk and height h. The moment of inertia with the 3 axis rotary axis or the n and n' axis is the sum of the moment of inertia of the ball being cut off, the tube and the cone backing, so that

$$I_{TT} = I_{bola} + I_{tabung} + I_{kerucut} \text{ [12]} \tag{3}$$

To calculate the moment of inertia each axis will be described in the calculation as follows.

Moment of Inertia Tippe Top at 3-axis

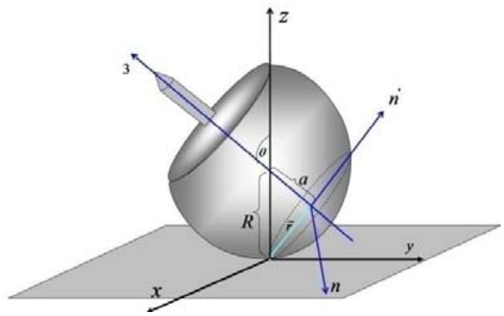

Figure 2. The rotary axis in the reverse top

The back casing consists of a solid ball that is cut off and a tube which is considered to be a handle on the return gasket [13]. The moment of inertia on the whole ball with the rotary axis on axis 3 is

$$I_{bola\ utuh} = \frac{2}{5}\ mR^2\ = \frac{8}{15}\rho\pi R^5 \tag{4}$$

with the mass of the ball equal to m=$\rho\frac{4}{3}\pi R^3$ [14] while the moment of inertia in the part of the backing ball is truncated by assuming the ball piece as a disk stack with certain limits that can be seen in the picture

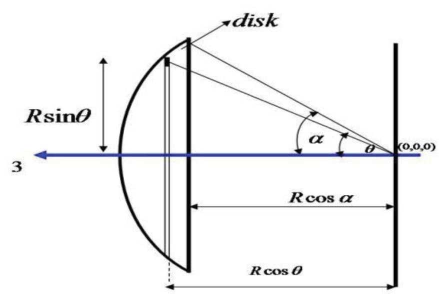

Figure 3. Cut back top ball with 3 axis as a rotary axis

The moment of inertia of the top back ball piece with the rotary axis is the 3 axis

$$dI = \frac{1}{2} dmr^2 \tag{5}$$

with

$$I_D = \frac{1}{2} m{R_D}^2 \tag{6}$$

and

$$R_D = R\sin\theta \tag{7}$$

that

$$\mathrm{dI} = \frac{1}{2} dm(R \sin\theta)^2 \tag{8}$$

with

$$I \;= \rho \int_{\theta=0}^{\alpha} \left[\frac{(R\sin\theta)^2}{2}\right] dV$$

$$= \frac{8}{15}\pi\rho R^5 + \left(\frac{1}{2}\pi\rho R^5 \left(\frac{4}{15}\sin^6\left(\frac{\alpha}{2}\right)(18\cos\alpha + 3\cos(2\alpha) + 19)\right)\right) \tag{9}$$

because

$$m_b = \rho \frac{4}{3}\pi R^3 \tag{10}$$

so

$$I_{bola\,TT} \;= m_b R^2 \left(\frac{32}{15} + \frac{2}{3}\left(\frac{4}{15}\sin^6\left(\frac{\alpha}{2}\right)(18\cos\alpha + 3\cos(2\alpha) + 19)\right)\right) \tag{11}$$

and for the moment of tube inertia on the backing handle with the 3 axis rotary axis is

$$I_{tabung} \qquad = \frac{1}{2} m_t b^2 [15] \tag{12}$$

whereas, the moment of cone inertia against axis 3 is

$$I_{kerucut} = \frac{3}{10} m_k {r_k}^2 \tag{13}$$

So, the moment of inertia of the backing on the 3 axis is

$$I_{TT} = \; I_{bola\,TT} + I_{tabung} + I_{kerucut}$$

$$= m_b R^2 \left(\frac{32}{15} + \frac{2}{3}\left(\frac{4}{15}\sin^6\left(\frac{\alpha}{2}\right)(18\cos\alpha + 3\cos(2\alpha) + 19)\right)\right) + \frac{1}{2} m_t b^2 + \frac{3}{10} m_k {r_k}^2.$$

(14)

Moment of inertia on n and n' axes

The back casing consists of a solid ball that is cut off and a tube which is considered to be the handle on the return gasket. The moment of inertia on the whole ball with the rotating axis at n or n' is

$$I_{bola\,utuh} = I_{CM} + I_d \tag{15}$$

$$= \frac{2}{5}\, mR^2 + ma^2 = \pi R^3 \left(\frac{16}{30}\rho R^2 + \frac{40}{30}\rho a^2\right) \tag{16}$$

while the moment of inertia in the part of the cut backing ball is:

In calculating the moment of inertia of the piece of the ball in the back, the backing ball piece is assumed to be a stack of disks with certain limits that can be seen in Figure 4

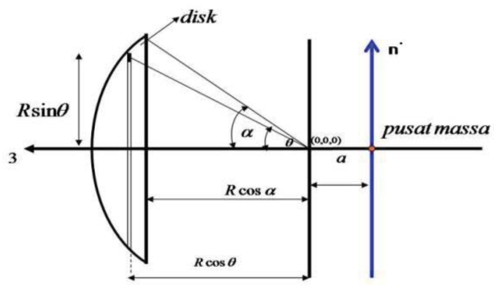

Figure 4. Cuted backing ball with n' as a rotary axis

thus, the moment of inertia of the top back ball piece can be calculated as follows

$$dI = I_D + dml^2 \tag{16}$$

with

$$I_D = \frac{1}{4} m{R_D}^2 \text{dan } R_D = R\sin\theta \tag{17}$$

And

$$dI = \frac{1}{4} dm(R\sin\theta)^2 + dm(R\cos\theta + a)^2 \tag{18}$$

So,

$$I = \rho \int_{\theta=0}^{\alpha} \left[\frac{(R\sin\theta)^2}{4} + (R\cos\theta + a)^2\right] dV \tag{19}$$

for the volume that covers the entire space is

$$dV = \pi {R_D}^2 dz \tag{20}$$

with

$$z = R\cos\theta \tag{21}$$

and

$$dz = dR\cos\theta - R\sin\theta\, d\theta \tag{22}$$

because R is constant, the first term is zero, so

$$dz = -R\sin\theta\, d\theta \tag{23}$$

So,

$$dV = \pi(R\sin\theta)^2(-R\sin\theta\, d\theta)$$

$$= -\pi R^3 \sin^3\theta\ d\theta \tag{24}$$

can be calculated the moment of inertia of the cut backing ball is

$$I\ = -\rho\int_{\theta=0}^{\alpha}\left(\frac{(R\sin\theta)^2}{4} + (R\cos\theta + a)^2\right)(\pi R^3\sin^3\theta\ d\theta)$$

$$= \pi R^3((-9\cos^5\alpha + 10\cos^3\alpha + 15\cos\alpha)R^2 - \frac{\begin{array}{c}(30\cos^4\alpha - 60\cos^2\alpha)aR + \\ (20\cos^3\alpha - 60\cos\alpha)a^2\end{array}}{60} + \frac{8R^2 + 15aR + 20a^2}{30}) \tag{25}$$

The moment of inertia of the truncated ball in the return top is the moment of inertia of the whole ball minus the moment of inertia of the return gasing ball piece

$$\mathrm{I}_{\text{bola TT}} = \mathrm{I}_{\text{bola utuh}} - \mathrm{I}_{\text{potongan bola TT}}$$

$$= \ \pi R^3\left(\frac{16}{30}R^2 + \frac{40}{30}a^2\right) - \pi R^3(-9\cos^5\alpha + 10\cos^3 + 15\cos\alpha)R^2$$

$$-\left(\frac{(30\cos^4\alpha - 60\cos^2\alpha)aR + (20\cos^3\alpha - 60\cos\alpha)a^2}{60} + \frac{8R^2 + 15aR + 20a^2}{30}\right)$$

$$= \pi R^3\left(\frac{16}{30}R^2 + \frac{40}{30}a^2\right) + \pi R^3(9\cos^5\alpha - 10\cos^3 - 15\cos\alpha)R^2$$

$$+\pi R^3\left(\frac{(30\cos^4\alpha - 60\cos^2\alpha)aR + (20\cos^3\alpha - 60\cos\alpha)a^2}{60} + \frac{8R^2 + 15aR + 20a^2}{30}\right)$$

$$= \pi R^3(\frac{24R^2 + 15aR + 60a^2}{30} + (9\cos^5\alpha - 10\cos^3\alpha - 15\cos\alpha)R^2 + \frac{(30\cos^4\alpha - 60\cos^2\alpha)aR + (20\cos^3\alpha - 60\cos\alpha)a^2}{60})$$

(26)

and for the moment of tube inertia on the backing handle with the rotary axis n and n' is

$$I_{tabung} = m_t(\frac{1}{4}b^2 + \frac{1}{12}(R - R\cos\alpha)^2$$

$$= \frac{1}{4}m_t\left(b^2 + \frac{1}{3}R^2(1-\cos\alpha)^2\right) \qquad (27)$$

whereas, the moment of cone inertia on the n and n 'axes, that is

$$I_{kerucut} = \frac{3}{5}m_k\left(\frac{{r_k}^2}{4} + {h_k}^2\right) + m_k(h_k + b - c + R\cos\alpha + a)^2 - m_k c^2 \qquad (28)$$

with hk is the TT cone height. So, the moment of inertia backing on the n and n’ axes is

$$I_{TT} = I_{bola\,TT} + I_{tabung} + I_{kerucut}$$

$$= \frac{3}{4}m_b\left(\frac{24R^2+15aR+20a^2}{30} + (9\cos^5\alpha - 10\cos^3\alpha - 15\cos\alpha)R^2 + \frac{(30\cos^4\alpha - 60\cos^2\alpha)aR + (20\cos^3\alpha - 60\cos\alpha)a^2}{60}\right) + \frac{1}{4}m_t b^2 + \frac{1}{3}R^2(1-\cos\alpha)^2 + \frac{3}{5}m_k\left(\frac{{r_k}^2}{4} + {h_k}^2\right) + m_k(h_k + b - c + R\cos\alpha + a)^2 - m_k c^2 \qquad (29)$$

CONCLUSION

Based on the analysis of the moment of inertia, back and forth through the Eular equation on the flat plane, the conclusion can be drawn that the moment of backing inertia

$$I_{TT} = I_{bola\,TT} + I_{tabung} + I_{kerucut}$$

$$= \frac{3}{4}m_b\left(\frac{24R^2+15aR+20a^2}{30} + (9\cos^5\alpha - 10\cos^3\alpha - 15\cos\alpha)R^2 + \frac{(30\cos^4\alpha - 60\cos^2\alpha)aR + (20\cos^3\alpha - 60\cos\alpha)a^2}{60}\right) + \frac{1}{4}m_t b^2 + \frac{1}{3}R^2(1-\cos\alpha)^2 + \frac{3}{5}m_k\left(\frac{{r_k}^2}{4} + {h_k}^2\right) + m_k(h_k + b - c + R\cos\alpha + a)^2 - m_k c^2$$

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