MATHEMATICS EDUCATION STUDENT'S REPRENTATION IN SOLVING ALGEBRA PROBLEMS BASED ON THEIR VARIOUS SPATIAL ABILITY

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Abstract

This paper aims to explore and describe the representation made by students of mathematics education in solving algebraic problems based on various spatial abilities. Two students from each of the high, moderate, and low spatial abilities were chosen to be the subject of the study. Data was obtained through in-depth interviews and written answers in answering algebraic problems. The credibility of the data was obtained through continuous observation and source triangulation. The data obtained were analyzed using a flow chart consisting of three activities that occurred simultaneously, namely data reduction, data presentation, and conclusion drawing. The analysis showed that the internal representation of subjects with various spatial abilities differed in understanding the problem and making a solution plan, both questions investigation and finding. The external representation of subjects with various spatial abilities is the same in understanding the problem of inquiry. As for the problem of finding high and low spatial-capable subjects presenting the same external representation. There are similarities in external representations for solving investigation questions between high and low spatial abilities, and between moderate and low spatial abilities for finding problems.

Keywords: Representation, Algebra Problems, Spatial Abilities.

INTRODUCTION

Algebra is one of the mathematics topics studied from elementary school to college. In addition algebra is widely used in everyday life, which is related to input, process and output. So it is not unusual for many of the educators and policy makers to claim that algebra is the gateway for success both in the world of education and the world of work. Therefore, student mastery of algebraic material is important. However, because algebra is an abstract part of mathematics, it is not difficult to understand. So it is necessary to declare the abstract to be more concrete with various representations. Someone has access to mathematical ideas only through the representation of the idea (Kilpatrick, Swafford, & Findell, 2001). This means that mathematical relationships, principles, and ideas can be expressed in various representations including visual representation (diagrams, drawings, or verbs), verbal representations (written and spoken languages), and symbolic representations (numbers, letters) (Panasuk and Beyranevand, 2011).

In addition, connecting different forms of representation in describing mathematics is an effective teaching strategy to reach students with different learning styles. In this context representation provides pedagogic strategies in mathematics learning especially algebra. Some researchers specifically look at the representations used to understand mathematics and a problem-solving strategy (NCTM, 2014). Goldin and Shteingold (2001) said that there are two systems of representation, internal and external representation systems. The system of internal representation is created in one's mind and is used to define mathematical meanings. In the context of solving internal representation problems in the form of problem-solving strategies, the problem-solving method keeps the internal components of the problem in mind (Bodner & Domin, 2000). External representation systems include conventional representations that are symbolic. External representation is something that represents, represents or represents objects and / or processes (Rosengrant, Van Heuleven, & Etkina, 2006). There are five models of external representation: contextual, visual, verbal, physical, and symbolic representation (NCTM, 2014; Huinker, 2015). Abrahamson (2006) suggests that mathematical representation is a conceptual composite, encompassing two or more connected ideas.

Problem solving is the most useful skill that students can use when they leave university. The problem that often arises is that students experience difficulties in solving problems that are unfamiliar with them. Students have difficulty in connecting concepts into mathematical information in meaningful ways and transferring conceptual aspects to the process of finding solutions (Kroesbergen & Van Luit, 2002; Olive & Steffe, 2002; Thevenot & Oakhill, 2008). Like most mathematics learning, algebra learning is focused on problem solving (NCTM, 2000). To overcome difficulties in learning algebra is by using various representations including using technology (Kieran & Yerushalmy, 2004; Van de Walle, 2010).It will be in conceptual composites, covering two or more connected ideas.

The problem-solving process has two stages, namely problem representation and problem solution (Krawec, 2010; Mayer, 2012). Representation of problems is the process of changing the problem description to the internal mental representation of problem solving through two stages: translate problems and integration. The translation of the problem is done by extracting the concept from the textual description of the problem by using linguistic and semantic knowledge. Linguistic knowledge is used to understand the meaning of words in textual descriptions, while semantic knowledge means factual knowledge in the world. Integration of problems is the process of connecting sentences in the problem description and resulting in a coherent representation. Once the problem description is translated into the internal mental representation of the problem solver, it means that he has understood the problem. This means the representation used in understanding the problem at Polya stage (1973). The problem solution is done by choosing a strategy or creating a plan and execution of the solution. Planning solutions are concerned with determining which operations to use and their sequence of uses. While the execution of the solution, relate in carrying out the calculations planned to solve the problem.

Some researchers review the representation and problem solving (Nasarudin, Lilia & Effandi, 2014, Boonen, van Wessel, Jolles, and van der Schoot, 2014; Krawec, 2012; Van Garderen, 2006) who examine the relationship of visual representation and performance of story completion (word problems). Diez mann & English (2001), Hall (2002) and Moseley (2005) examined the use of representation when studying mathematical concepts and solving problems that focused on students' ability to produce and use certain forms of representation. Ozgun-Koca (2001) and Panasuk and Beyranevand (2011) conclude that students have a preference for using certain types of representation when solving algebraic problems in general, and that preference is related to the level of students' ability in mathematics. These researchers all reviewed the representation of elementary school students based on mathematical ability. Baiduri (2018) examines the various external representations of first semester mathematics education students when solving problems and it is associated with the correctness of the answers.

Based on the facts that he importance of representation in learning and problem solving of mathematics, the research with the focus of representation by observing the various characteristics of the subject is still very open. Therefore, in this paper, we examine the representation in problem solving based on spatial ability. Spatial ability is the ability to understand and remember the spatial relationships between objects (Allen, 2003) that include object identification (answer the question "What is it?"), The location of the object (answer the question "Where is it?"), And the travel orientation (answer the question "Where am I?"). The ability to visualize and use spatial reasoning is seen as a key component of reasoning, problem solving, and building accurate evidence of mathematical phenomena (Arcavi, 2003). As a collection of cognitive skills that enable a person to interact with his environment, spatial ability has been a field of study for decades (Hegartydan Waller, 2005) and it is crucial to improving success in mathematics and science, which is one of the most important subjects, successful in engineering work (Halpern, 2000).

METHOD

2.1 Subjects

To select subjects with various spatial abilities, 44 first semester mathematics students of University of Muhammadiyah Malang, East Java, Indonesia academic year 2017/2018 were given spatial ability tests. The test results obtained the lowest score of 44 and the highest 93, mean (Me) = 77 and standard deviation (SD) = 11.64 in the range 0 - 100. Furthermore the test scores are grouped into three categories. High category if score≥Me + SD, moderate if Me-SD <score <Me + SD and low if score ≤ Me-SD. Furthermore from each category taken two students who can communicate well and they become the subject of research.

2.2. Instruments

The research instrument consists of the main instrument, the researcher himself and the supporting instrument, the audio visual recorder, spatial ability test, algebra problem solving test, and interview guide.

2.2.1. Spatial Capability Test

About the spatial capability test in the adaptation of Newton and Bristoll (copy right www.psychometric-success.com), ie by translating English into Indonesian. A total of 45 questions are presented with the time provided to answer for 20 minutes.

2.2.2.Algebra Problem-Solving Task

The task problem consists of two types of questions, one problem of proof / investigation and one finding (Polya, 1973) adopted from Baiduri (2018). The answer to this question allows the subject to use various representations.

Problem I. Investigate whether the three lines $y = 2x - 1$; $y = x - 1$; $y = 4x - 5$ through a point?

Problem II. Compute the lengths of the right triangle sides which are consecutive integers!

2.2.3. Interview guidelines

An interview guide was developed to help reveal the subject representation process when solving algebraic problems, especially internal representations.

2.3. Data Collection and Its Analysis

Data collection begins by asking the subjects to complete the troubleshooting test. After completion with an in-depth interview (semi-structured interview), the data is recorded with a video recorder. Then to ensure the credibility of the data obtained, the researchers made continuous observations and triangulation of sources (Moleong, 2011; Sugiyono, 2011). Furthermore, credible data in the analysis with interactive models, three-flow activities carried out simultaneously: data reduction, data presentation and conclusion (Miles & Huberman, 1992).

RESULTS AND DISCUSSION

Representation made by the subject in problem solving is divided into several stages, namely the stage of understanding the problem, planning and implementing the settlement, and checking the answers based on the type of problem. Before the subject works, an in-depth interview is conducted related to the stages of problem solving.

3.1 Representation of Problem Solving Inquiry (Problem I)

3.1.1. Students with high spatial abilities

Based on the interview, the subject in understanding the problem by doing textual description of the problem related to what was given and asked on the problem in representation translates the problem (Krawec, 2010; Mayer, 2012) or visual image (visual imagery) (van Garderen, 2006). The textual description of what is known by mentioning three lines, while the one being asked states whether the three lines go through a point. Externally the representation is text / verbal and symbolic stated in Figure 3.1.1

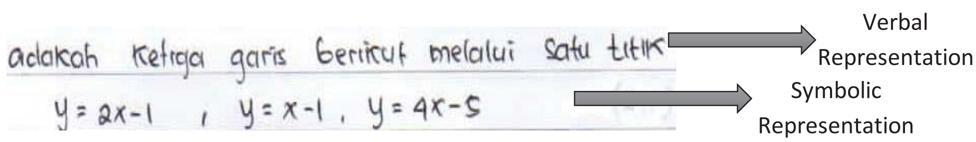


Figure 3.1.1. External representation of subjects with high spatial ability understands problem I

After understanding the problem, then to solve the problem the subject illustrates in his mind that the strategy will be carried out to answer the question by looking for the points of cutting the three lines analytically. Furthermore, in his mind, he explained that to find the intersection with substitution or elimination methods. Furthermore, if the three intersection points are equal through one point. Conversely, if not the same means the three lines do not go through a single point. Means the subject does the process of connecting his understanding of the problem, concept and count operation that will be used in solving the problem. In the context of representation in the problem solving process the subject has integrated the problem (Krawec, 2010; Mayer, 2012) or schemata (Jitendra, 2002). In writing the subject work is stated in Figure 3.1.2.

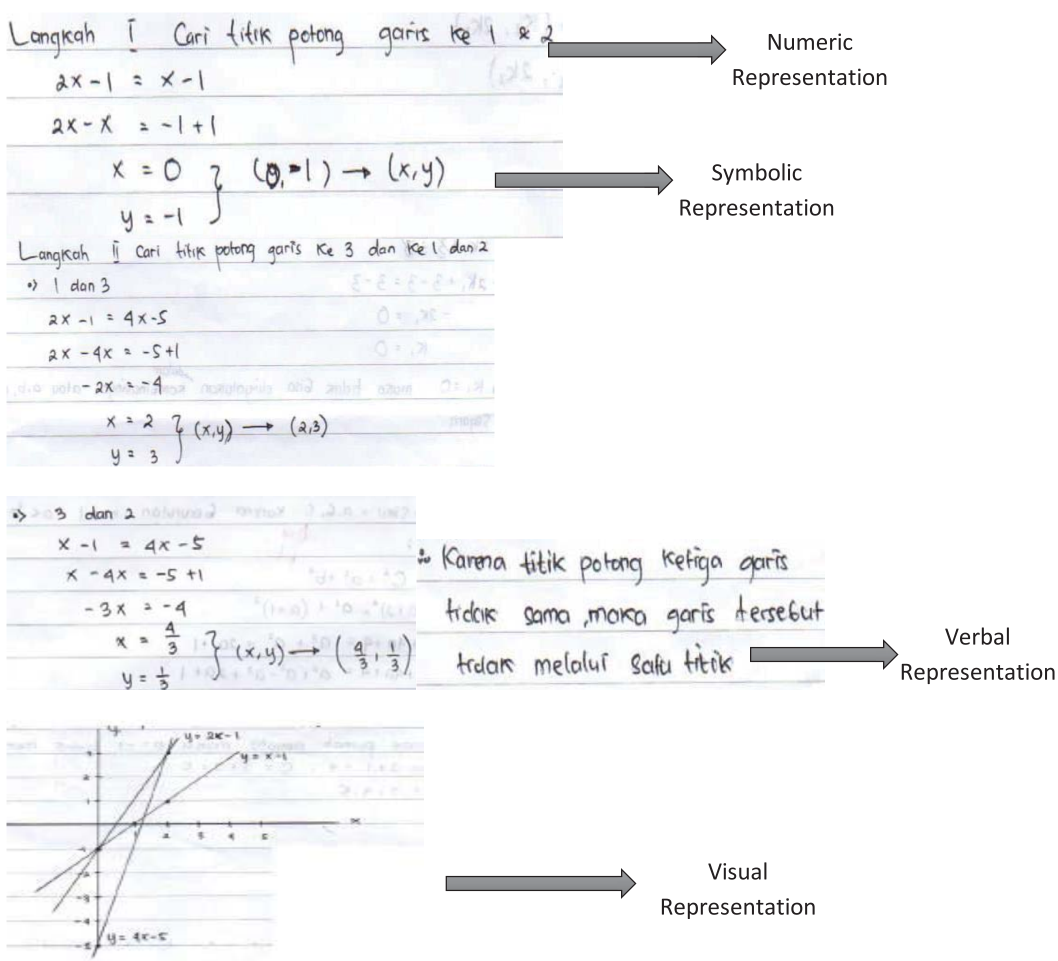


Figure 3.1.2. Representation of external subjects with high spatial ability to solve problem I

The subject's work shows the use of various external representations when completing questions, namely: 1) symbolic, using notation (x, y) to describe a point in the Cartesian plane, 2) verbal, use of statements to draw conclusions from a work, 3) numerical, when looking for cut points, and 4) visual, by describing the graph of the three lines given (Bal, 2014; NCTM, 2014; Huinker, 2015).

Furthermore based on the interview, the subject has examined the work which includes the count operations used and the results when completing and has other alternatives how to solve the problem by describing the graph of the three lines. The subject also stated that the solution in a graphical (visual) way is more practical when compared analytically. This is because the graphical solution (visual) results can easily be seen, right or wrong. This is in line with the results of previous studies which stated that the solution becomes more successful if using visual representation (Arcavi, 2003; Boonen, van Wesel, Jolles, & van der Schoot, 2014; Cankoy and Özder, 2011).

3.1.2. Students with moderate spatial abilities

After reading the question, then based on the interview the subject explains that there are three lines then the question is whether the three lines go through a point. External representation given by the subject by rewriting the question and giving a notation of each line with letters (a), (b), and (c) as in Figure 3.1.3. Giving notation for each line equation shows that the subject has textual description of the problem to his understanding of the problem. In the context of internal representation means the subject has translated the problem (Krawec, 2010; Mayer, 2012) or visual imagery (visual imagery) (van Garderen, 2006). External representation carried out by the subject when understanding the problem is a verbal and symbolic representation.

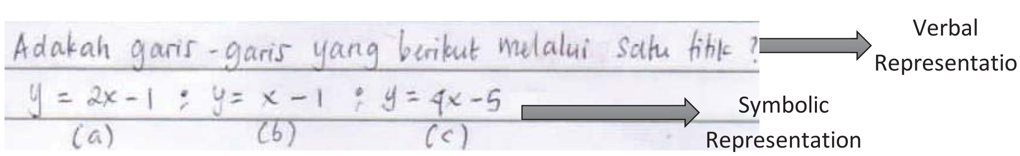


Figure 3.1.3. Representation of external subjects with moderate spatial ability understand problem I

At the planning and completion stage, the subject explains the strategy that will be used by: 1) finding the intersection of two lines, (a) and (b) by elimination and substitution, 2) substituting the intersection point of result 1) in line (c). If the result of the left segment is the same as the right segment, then the three lines go through a point. Conversely if not the same means the three lines do not go through one point. This means that it has connected what elements are known and asked about the problem, how to find a solution and draw conclusions about the results of work in the problem solving process, which in the context of the representation of the subject has integrated the problem (Krawec, 2010; Mayer, 2012). External representations carried out by the subject when solving problems are symbolic and verbal representations (Bal, 2014; NCTM, 2014; Huinker, 2015) or schemata (Jitendra, 2002). Symbolic representation is done when the problem solving process is numerical when looking for intersection points, while verbal representation is done when concluding the work results. This representation is presented in Figure 3.1.4.

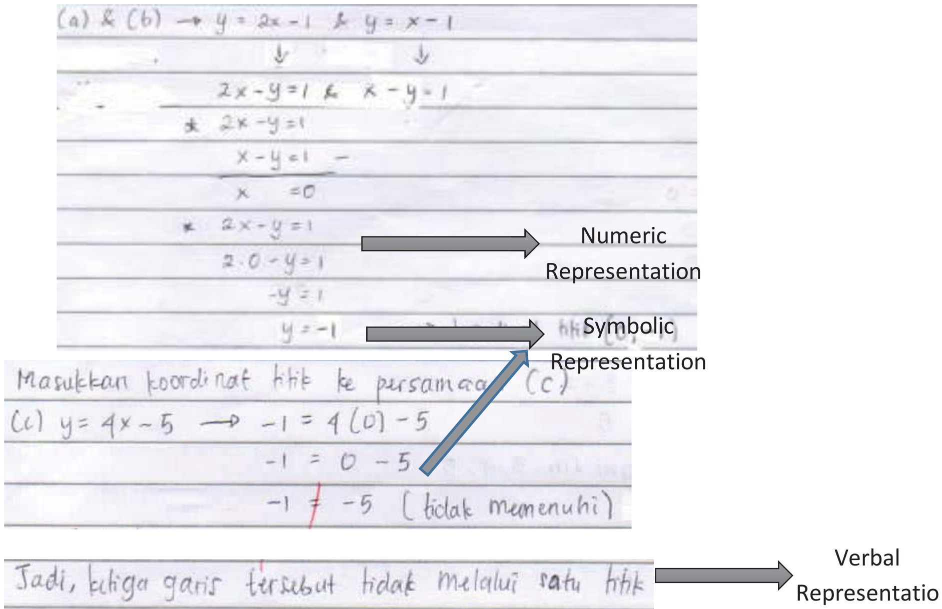


Figure 3.1.4. Representation of external subjects with moderate spatial ability to solve problem I

3.1.3. Students with low spatial abilities

Understanding the problem of the subject states there are three lines each called a), b) and c) and the question which line goes through one point. This is in accordance with external representations that are presented in written form, in the form of symbolic and verbal representations. When mentioning which three lines and lines are through one, the subject describes textually what is known and what is asked about the problem. However, the description of what was asked was still inappropriate. The process of textual description of the problem in the context of representation, the subject has translated the problem (Krawec, 2010; Mayer, 2012) but failed to do visual imagery (van Garderen, 2006) when understanding what was asked. Understanding the subject of the problem is represented symbolically and verbally as in Figure 3.1.5.

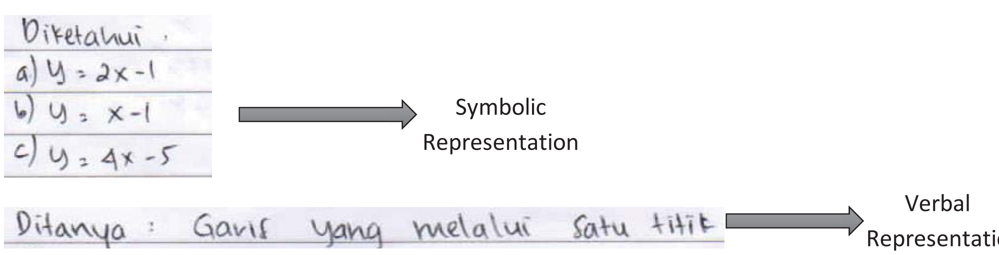


Figure 3.1.5. Representation of external subjects with low spatial ability to understand problem I

At the planning stage of the results of the interview, the subject explains the strategy used in answering the question is 1) by describing each line, 2) looking for the intersection of two lines, a) and and a line b). How to describe each graph by taking two points, namely the point on the X axis and the point on the Y axis. While to find the intersection of the two lines with substitution and elimination. Means that the subject has integrated the problem (Krawec, 2010; Mayer, 2012) or schemata (Jitendra, 2002) by connecting his understanding of problems, concepts and procedures to be used in problem solving. The written answer to the subject is presented in Figure 3.1.6.

Based on the answers written in Figure 3.1.6, the external representation conducted by the subject (Bal, 2014; NCTM, 2014; Huinker, 2015) is 1) symbolic, when looking for the point of intersecting lines with the X and Y axes, 2) visual, when the subject draw a graph of the equation of the line, 3) numerical, when looking for intersections, and 4) verbal, when concluding a line through two points.

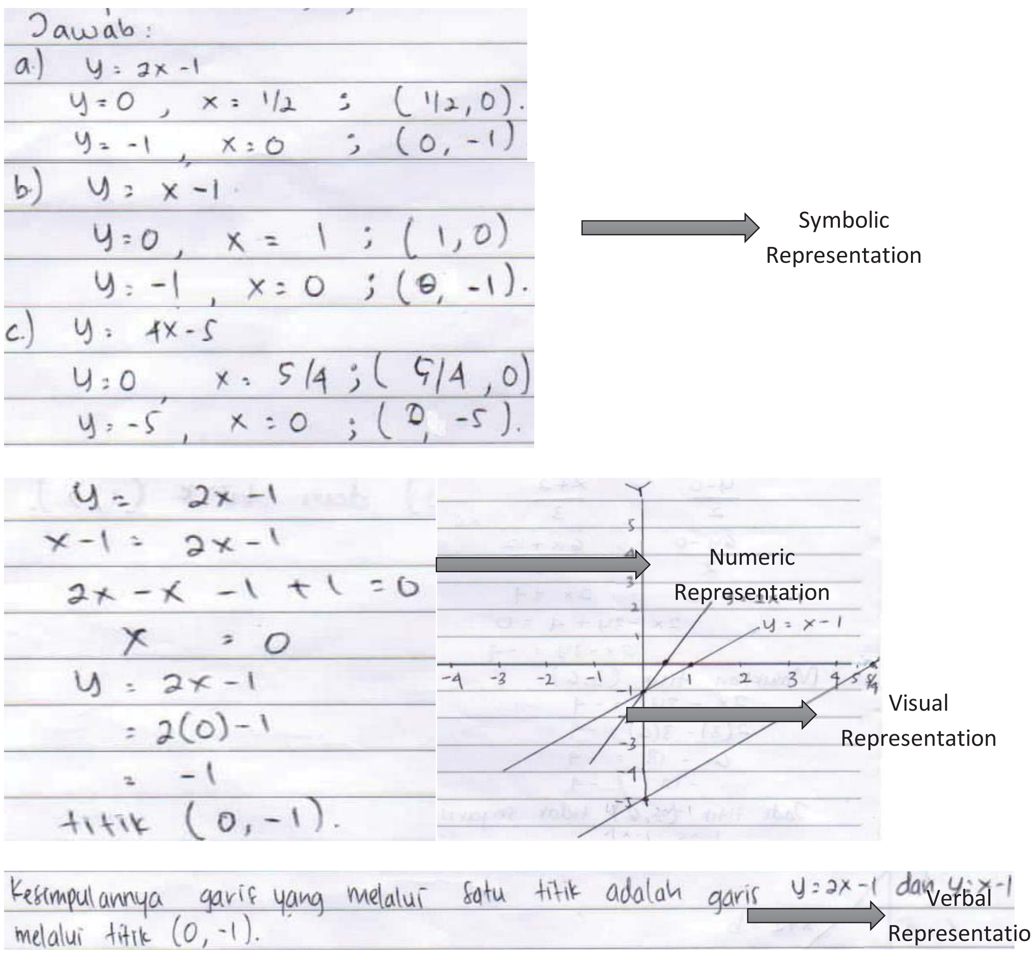


Figure 3.1.6. Representation of external subjects with low spatial ability to solve problem I

3.2 Finding Problem Resolution Representation (Problem II)

3.2.1. Students with high spatial abilities

The results of interview show that the subject understanding the problem by equating the right triangle a, b, and c with a < b < c. This is based on the three sides being three consecutive integers. What is the number problem? This shows that the subject textual description of the problem related to what is given is a right triangle, which has three sides and sides are three sequential integers in the context of subject representation doing translation of the problem (Krawec, 2010; Mayer, 2012) or visual imagery (visual imagery) (van Garderen, 2006). In addition the subject connects the concept of the length of the side of a triangle with three consecutive numbers (a < b < c) which in the context of representation in problem solving the subject integrates problems (Krawec, 2010; Mayer, 2012). Externally, the representation is text / verbal and symbolic that stated in Figure 3.2.1.

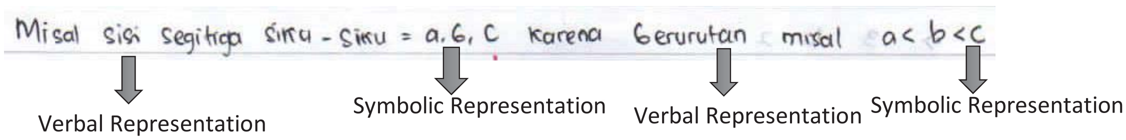
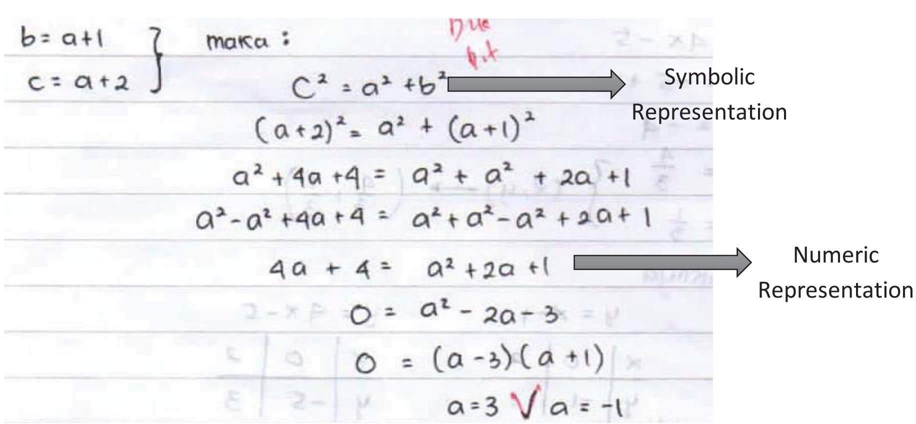


Figure 3.2.1. External representation of subjects with high spatial ability understands problem II

After understanding the problem, to solve the problem the subject associates the side relationship in the form of a sequential integer, where the sides are a, b = a +1 and c = a + 2. Then look for the amusing value of the Pythagorean theorem, $c^2 = a^2 + b^2$, where a, b and c must not be negative, because the length of the side of the triangle means that in making the completion plan the subject connects the strategy with its understanding of the problem, the triangular concepts, sides, and the Pythagorean theorem that will be used in problem solving. If linked representation in the process of solving problems, the subject has integrated the problem (Krawec, 2010; Mayer, 2012) or schemata (Jitendra, 2002). In writing the subject work is stated in Figure 3.2.2.



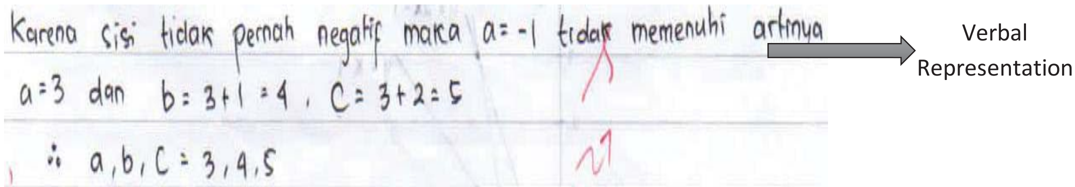


Figure 3.2.2. Representation of external subjects with high spatial to solve problem II

The subject's work shows the use of various external representations when completing questions, namely: 1) symbolic, using triangle side length notation, 2) numerical, when looking for the length of one side using the Pythagorean theorem, and 3) verbal, the use of statements to draw conclusions from a job (Bal, 2014; NCTM, 2014; Huinker, 2015).

3.2.2. Students with moderate spatial abilities

Based on the interview, the subject explained that in the question given a right triangle, the sides are sequential integers. Whereas those numbers are requested. External representation given by the subject by rewriting the question as shown in Figure 3.2.3. External representation that the subject does when understanding the problem is a verbal representation.

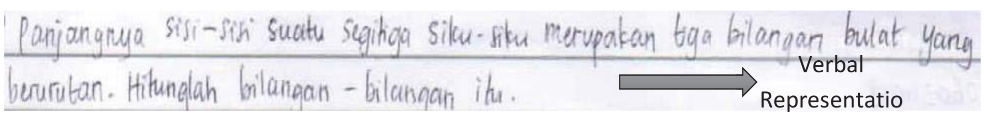
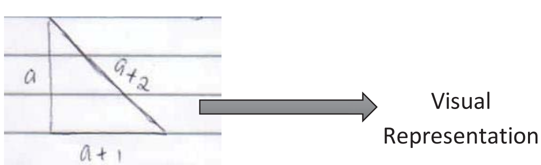


Figure 3.2.3. Representation of external subjects with moderate spatial ability to understand problem II

Before solving the problem, the subject explains the strategy that will be used by specifying the side of the triangle with a, a + 1, and a + 2. Then to find the value of a is used the Pythagorean theorem. This is because the triangles are right. This means that it has connected what elements are known and asked about the problem, the concept that applies to right triangles in the problem solving process, which in the context of representation of the subject has integrated the problem (Krawec, 2010; Mayer, 2012) or schemata (Jitendra, 2002). External representations made by the subject when solving problems are symbolic, visual and verbal representations (Bal, 2014; NCTM, 2014; Huinker, 2015). Symbolic representation is done when the problem solving process, visual representation is done when describing a right triangle, numerically when looking for the length of one side using the Pythagorean theorem, while verbal representations are made when drawing conclusions. Completion of the subject is presented in Figure 3.2.4.



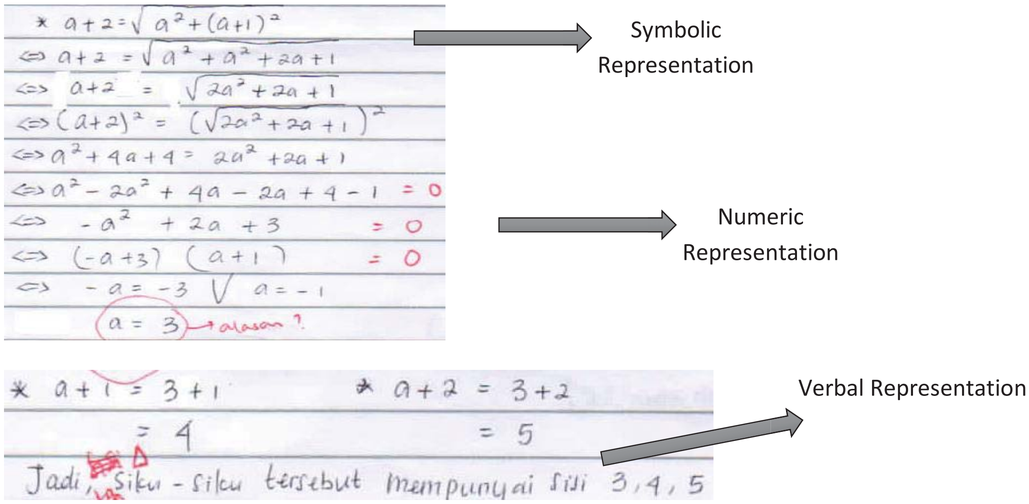


Figure 3.2.4. Representation of external subjects with moderate spatial ability solve problem II

3.2.3. Students with low spatial abilities

The results of the interview show that the subject in understanding the problem by equating the sides of a right triangle a, b, and c with a = x, b = x + 1, and c = x + 2. This is based on the three sides are three integers in sequence. What is the number problem? This shows that the subject textual description of the problem is related to what is given a right triangle, which has three sides and sides are three consecutive integers which in the context of subject representation do the translation of the problem (Krawec, 2010; Mayer, 2012) or visual image (visual imagery) (van Garderen, 2006). Externally the representation is text / verbal and symbolic stated in Figure 3.2.5.

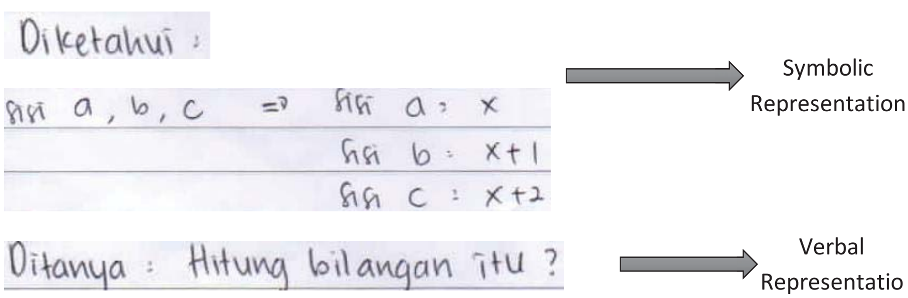


Figure 3.2.5. Representation of external subjects with low spatial ability understand problem I

Based on the results of the interview, before the problem solving is done, the strategy will be used by the subject by describing a right triangle by assuming the sides of the triangle x = a, c = x + 1, and b = x + 2. Next to find the value of x used theorem Pythagoras. This is because the triangles are right. After obtaining the value of x, then by substituting the values of a, b, and c. This means that it has connected what elements are known and asked about the problem, the concept that applies to right triangles in the problem solving process, which in the context of the representation of the subject has integrated the problem (Krawec, 2010; Mayer, 2012). External representations made by the subject when solving problems are symbolic, visual, numerical, and verbal representations (Bal, 2014; NCTM, 2014; Huinker, 2015). Symbolic representation is done when the problem solving process, visual representation is done when in describing a right triangle, numeric, when looking for the length of one side using the Pythagorean theorem, while verbal representation is done when drawing conclusions related to the x value which is the solution of the given problem. Completion of the subject is presented in Figure 3.2.6.

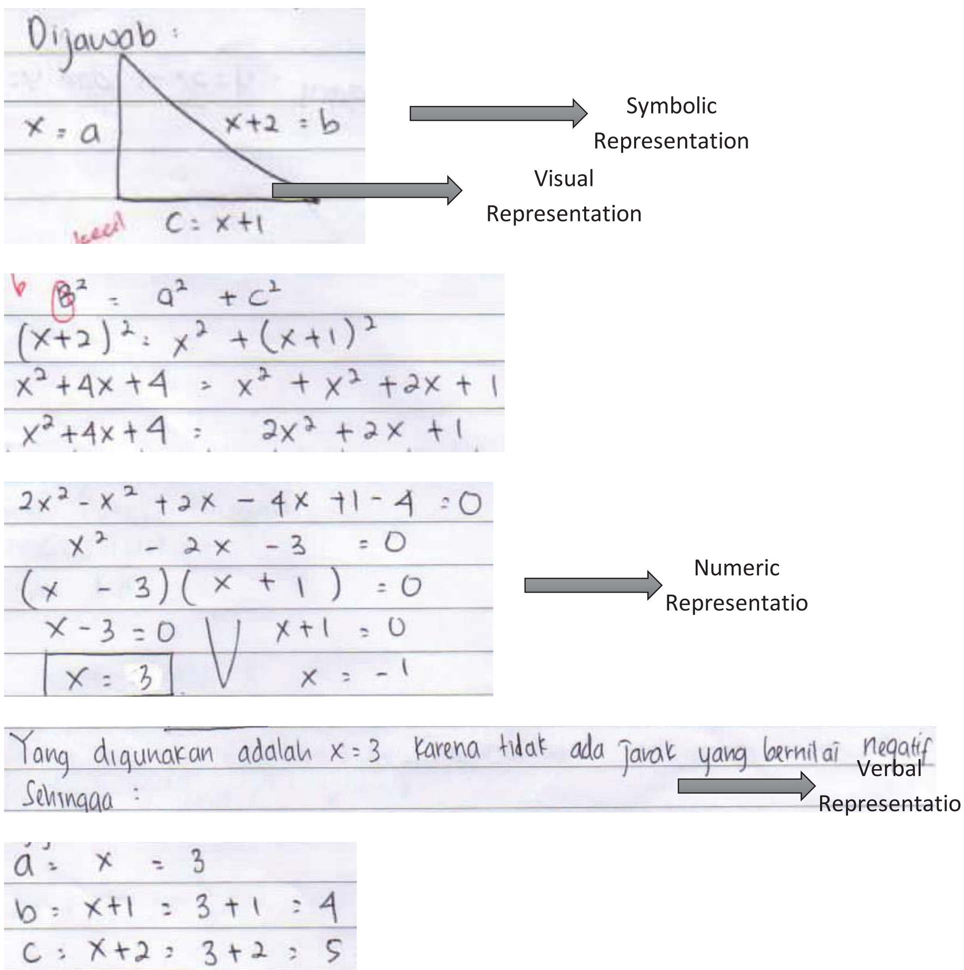


Figure 3.2.6. Representation of external subjects with low spatial ability solve problem II

CONCLUSION AND REMARK

Problem solving is an important skill that helps individuals deal with various daily problems in living their lives. Therefore educators (teachers) need to prepare their students in this regard by paying attention to various characteristics of their abilities. This paper describes various representations, both internal and external students of mathematics education in solving the problem of investigation and discovery (Polya, 1973) based on their spatial abilities.

For the problem of investigation, the external representation made by the subject with various spatial abilities in understanding the problem is the same, namely verbal and symbolic representation. However, external representation when solving differences occurs between those with high and low spatial abilities and those with moderate spatial abilities. Subjects with high and low spatial abilities when solving problems do various representations, namely verbal, symbolic, numerical, and visual. Whereas the spatial-capable person is doing three representations, not doing visual representation

In the problem of finding, verbal and symbolic external representations are carried out by high and low spatial-capable subjects when understanding problems, while those with low spatial abilities make external verbal representations only. When the subject is moderate and low spatial ability to solve the problem of carrying out various representations, namely verbal, symbolic, numerical, and visual. While those with high spatial ability do three representations, do not do visual representation.

The internal representation of each subject is different in understanding the problem and in making settlement plans for both the problem of investigation and finding. This means that the thinking process in solving problems is influenced by spatial abilities.

This study uses the subject of mathematics education students in year I by using the problem of investigation and finding. Further research needs to be studied for this type of contextual problem with the subject of elementary level students or further year students (years III or IV). In addition, it is necessary to examine the relationship between representation capabilities, spatial ability, and mathematical problem solving.

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