DESIGN-BASED RESEARCH IN MATHEMATICS EDUCATION[1]

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Abstract

This paper sketches the approach of design-based research in mathematics education with results of two innovation-oriented projects. The projects investigated how students can be involved in the development of mathematical concepts and skills by using design tools related to guided reinvention and emergent modelling. Both studies combine design-based research with a prominent role for the hypothetical learning trajectory as a research instrument in the three phases of design research (design, teaching experiment, retrospective analysis). Each of the phases of the research cycles is addressed: the preliminary phase in which a hypothetical learning trajectory and instructional activities are designed, the teaching experiment phase and the phase of the retrospective analysis. We conclude with a reflection on design-based research as an approach to study innovative teaching approaches that offers researchers to take into account contextual factors and that create opportunities for others to adapt the results to their research or teaching practice.

1. Introduction

One of the salient characteristics of mathematics is the use of symbols. This is not merely an external characteristic, as mathematical symbols are an integral part of mathematics. It is hard to think about measurement without the use of unit measures, or to understand calculus without pointing to rates of change in graphs. This intertwinement of meaning and visual representations poses a problem for mathematics education. Experts - like teachers and instructional designers - tend to

[1] This paper is an adaption from: Bakker, A., Doorman, M., & Drijvers, P. (2003). *Design research on how IT may support the development of symbols and meaning in mathematics education*. Paper presented at the Onderwijs Research Dagen (ORD), Kerkrade, The Netherlands.

see these symbols as carriers of meaning. For them, symbols and graphs are transparent; they can "see the mathematics through it", so to speak. The students, however, often do not have the necessary mathematical background to interpret those symbolic representations in that manner. As a consequence, teachers will have to explain to the students what there is to see, and how to reason with those symbolic representations. This, Cobb et al. (1992) point out, leads to proceduralising and algorithmetising and the loss of meaning - or to, as van Oers (2000) calls it, "pseudo mathematics".

To find a way out of this dilemma, one may consider the history of mathematics to investigate how meaning and symbols emerged. It turns out that mathematical symbols did not arrive ready-made, with full-fletched meaning. Instead, one can discern a reflexive process in which symbolising and the development of meaning co-evolve (Meira, 1995). Symbolising, here, refers to inventing and using a series of symbols. In relation to this, Latour (1990) and others (e.g. Roth & McGinn, 1998) speak of a "cascade of inscriptions". This notion of a cascade of inscriptions has its counterpart in semiotic concepts as "chain of signification" (Walkerdine, 1988; Whitson, 1997).

The current challenge for mathematics educators is to develop mathematics education that is in line with these dynamic conceptions of symbolising and development of meaning. The task of researchers is to shed light on the key elements of this type of mathematics education. In order to investigate the possibilities of such a new and innovative approach to mathematics education is that the instructional materials are not available yet. Moreover, research into the topic requires a process in which the design of instructional activities and teaching experiments are intertwined with the development of instructional theories for specific topics in mathematics. In this paper we will discuss two projects with the aim to illustrate the characteristics of such a design-based research approach that ensure a systematic approach and that offer the opportunities to generalize findings over specific contexts. For each project, this implies a dual goal:

- on the one hand, answering the question on how to develop and investigate an innovative teaching and learning process, and
- on the other hand, investigating the reflexive relations between symbol use and the development of meaning.

Given these goals, we chose what is called design-based research or developmental research (Gravemeijer, 1994, 1998), as our research method. Following Brown (1992), Cobb, Confrey, diSessa, Lehrer and Schauble (2003) refer to this type of research as design experiments, which they elucidate in the following manner:

> Prototypically, design experiments entail both "engineering" particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them. This designed context is subject to test and revision, and successive iterations that result play a role similar to that of systematic variation in experiment.
> (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003, p. 9)

In this description, the two central aspects of this paper come to the fore in (a) the design of means of support for particular forms of learning, and (b) the study of those forms of learning. In each of the two research projects under discussion, the backbone of the design is formed by the design, development and revision of a hypothetical learning trajectory.

2. Design-based Research

The design-based research approach has a cyclic character in which thought experiments and teaching experiments alternate. A cycle consists of three phases: the preliminary design phase, the teaching experiment phase, and the phase of retrospective analysis. A second characteristic of design-based research is the importance of the development of a learning trajectory that is made tangible in instructional activities. The design of instructional activities is more than a necessity for carrying out teaching experiments. The design process forces the researcher to make explicit choices, hypotheses and expectations that otherwise might have remained implicit. The development of the design also indicates how the emphasis within the theoretical development may shift and how the researcher's insights and hypotheses develop. As Edelson argues, design is a meaningful part of the research methodology:

(...) design research explicitly exploits the design process as an opportunity to advance the researchers understanding of teaching, learning, and educational systems. Design research may still incorporate the same types of outcome-based evaluation that characterise traditional theory testing, however, it recognizes design as an important approach to research in its own right. (Edelson, 2002, p.107)

This is particularly the case when the theoretical framework involved is under construction.

2.1. Hypothetical learning trajectory

Within each macro level research cycle, we distinguish three phases: the preliminary design phase, the teaching experiment phase, and the phase of retrospective analysis. The first phase of preliminary design includes two related parts, the development of a Hypothetical Learning Trajectory (HLT) and the design of instructional activities. In the next four sections we elaborate on each of these (partial) phases.

The first phase of each research cycle includes the development of a "Hypothetical learning trajectory" – a term that is taken from Simon (1995). Originally, Simon used the HLT for designing and planning short teaching cycles of one or two lessons. In our study, however, we developed HLTs for teaching experiments that lasted for sequences varying from 8 to 20 lessons. As a consequence, the HLT comes close to the concept of a local instruction theory (Gravemeijer, 1994). A second difference with Simon's approach is that Simon took a teacher's perspective, whereas we take a researcher's perspective.

The development of an HLT involves the choice or design of instructional activities in relation to the assessment of the starting level of understanding, the formulation of the end goal and the conjectured mental activities of the students. Essential in Simon's notion of a HLT is that it is hypothetical; when the instructional activities are acted out, the teacher – or researcher in our case – will be looking for evidence of whether these conjectures can be verified, or should be rejected.

For the design of the student activities, their motivation and the estimation of their mental effects, the designer makes full use of his domain specific knowledge, his repertoire of activities and representations, his teaching experience, and his view on the teaching and learning of the topic. After a field test by means of a teaching experiment, the HLT will usually be adapted and changed. These changes, based on the experiences in the classroom, start a new round through the mathematical teaching cycle, and, in terms of the design research approach, the next research cycle.
The concept of the HLT may seem to suggest that all students follow the same learning trajectory at the same speed. This is not how the HLT should be understood. Rather than a rigid structure, the HLT represents a learning route that is broader than one single track and has a particular bandwidth.

With an emphasis on the mental activities of the students and on the motivation of the expected results by the designer, the HLT concept is an adequate research instrument for monitoring the development of the designed instructional activities and the accompanying hypotheses. It provides a means of capturing the researcher's thinking and helps in getting from problem analysis to design solutions.

2.2. Design of instructional activities

The preliminary design phase of the design research cycles includes the development of the HLT and the instructional activities. The expectations of the students' mental activities established in the HLT are elaborated into specific key activities in the instructional materials.

The design of instructional activities in these studies included the development of student text booklets and teacher guides. While designing these materials, choices and intentions were captured and motivated, to inform the teacher and to keep track of the development of the designer's insights. When the materials were about to be finalised, these aims and expectations were described at the task level. Key items, that embodied the main phases in the HLT, were identified. These items reflected the relevant aspects of the intended learning process and were based on the conceptual analysis of the topic. The identification of key items guided observations and prepared for the retrospective data analysis. Finally, teacher guides as well as observation instructions were written, to make intentions and expectations clear to teachers and observers. During the design phase, products were presented to colleagues, teachers and observers. This led to feedback that forced the researcher to become explicit about goals and aims, and that provided opportunities for improving all the materials.

While designing instructional activities, the key question is what meaningful problems may foster students' cognitive development according to the goals of the HLT. Three design principles guided the design process: guided reinvention, didactical phenomenology and emergent models.

The design principle of *guided reinvention* involves reconstructing the natural way of developing a mathematical concept from a given problem situation. A method for this can be to try to think how you would approach a problem situation if it were new to you. In practice, this is not always easy to do, because as a domain expert it is hard to think as if you were a freshman. The history of the domain can be informative on specific difficulties concerning concept development (e.g. Gravemeijer & Doorman, 1999).

The second design principle, *didactical phenomenology*, was developed by Freudenthal (Freudenthal, 1983). Didactical phenomenology aims at confronting the students with phenomena that "beg to be organised" by means of mathematical structures. In that way, students are invited to build up mathematical concepts. Meaningful contexts, from real life or "experientially real" in another way, are sources for generating such phenomena (de Lange, Burrill, Romberg, & van Reeuwijk, 1993; Treffers, 1987). The question, therefore, is to find meaningful problem contexts that may foster the development of the targeted mathematical objects. The context should be perceived as natural and meaningful, and offer an orientation basis for mathematisation.

The last remark leads to the third design principle, the use of *emergent models* (Gravemeijer e.a., 2000; Van den Heuvel-Panhuizen, 2003). In the design phase we try to find problem situations that lead to models that initially represent the concrete problem situation, but in the meantime have the potential to develop into general models for an abstract world of mathematical objects and relations.

2.3. Teaching experiments

The second phase of the design research cycle is the phase of the teaching experiment, in which the prior expectations embedded in the HLT and the instructional activities are confronted with classroom reality. The term "teaching experiment" is borrowed from Steffe (Steffe & Thompson, 2000). The word "experiment" is not referring to an experimental group - control group design. In this section we explain how the teaching experiments were carried out; in particular, we pay attention to the data sampling techniques used during the teaching experiments.

The research questions share a process character: they concern the development of understanding of mathematical concepts. Therefore, we focussed on data that reflected the learning process and provided insight into the thinking of the students. The main sources of data were observations of student behaviour and interviews with students. The observations took place on three levels: classroom level, group level and individual level. Observations at classroom level concerned classroom discussions, explanations and demonstrations that were audio and video taped. These plenary observations were completed by written data from students, such as handed in tasks and notebooks.

Observations at group level took place while the students were working on the instructional activities in pairs or small groups. Short interviews were held with pairs of students. In addition to this, the observers made field notes.

The lessons were evaluated with the teachers. In particular, the organisation of the next lesson and the content of the plenary parts were discussed. Also, decisions were taken about skipping (parts of) tasks because of time pressure. Such decisions were written down in the teaching experiment logbook.

2.4. Retrospective analysis

The third phase of a design research cycle is the phase of retrospective analysis. It includes data analysis, reflection on the findings and the formulation of the feed-forward for the next research cycle.

The first step of the retrospective analysis concerned *elaborating on the data*. A selection from video and audiotapes was made by event sampling. Criteria for the selection were the relevance of the fragment for the research questions and for the HLT of the teaching experiment in particular. Data concerning key items was always selected and these selections were transcribed verbatim. The written work from the students was surveyed and analysed, especially the work on key items, tests and hand-in tasks. Results were summarised in partial analyses. This phase of the analysis consisted of *working through the protocols* with an open approach that was inspired by the constant comparative method (Glaser & Strauss, 1967; Strauss & Corbin, 1988). Remarkable events or trends were noted as conjectures and were confronted with the expectations based on the HLT and the instructional activities.

The second phase of analysis concerned *looking for trends* by means of sorting events and analysing patterns. The findings were summarised illustrated by prototypical observations. These conjectures were tested by surveying the data to find counterexamples or other interpretations, and by data triangulation: we analysed the other data sources, and in particular the written student material, to find instances that confirmed or rejected the conjectures. Analysis of the written materials often evoked a reconsideration of the protocols. Analysis was continued in this way until saturation, which meant that no new elements were added to the analysis and no conclusions were subject to change.

The third phase in analysing the data was the *interpretation of the findings* and the comparison with the preliminary expectations of the HLT. Also, explanations for the differences between expectations and findings were developed. These conclusions and interpretations functioned as feed-forward for the formulation of new hypotheses for the next cycle in the research.

3. An exemplary case: the basic principles of the mathematics of change[2]

Background and design of HLT

The aim of this project is to find out how students can learn the basic principles of calculus and kinematics by modelling motion. Nowadays, graphs are used in calculus and kinematics education as representations for describing change of velocity or distance travelled during a time interval. Students are expected to give meaning to the relation between distance travelled and velocity through characteristics of these graphs such as area and slope. The use of such instructional materials is based on a representational view (Cobb et al., 1992), which assumes that instructional materials can represent scientific knowledge, and that scientific concepts can be made accessible without fully taking into account the limitations of the knowledgebase of the students into which they have to be integrated.

Cobb et al. oppose this view. In line with their reasoning, we claim that symbolisations and knowledge of motion can co-evolve in a learning process. Theories on symbolising give rise to heuristics for designing a learning route within which the mathematical and scientific knowledge emerges from the activity of the students (Gravemeijer et al. 2000). In this route, the creation, use and adaptation of various graphical representations are interwoven with learners' activities in a series of science-practices, from modelling discrete measurements to reasoning with continuous models of motion. Our focus is on students' contributions during these practices, and how we can built upon their contributions towards the intended attainment targets. Consequently, for understanding their reasoning we use the design-based research approach of planning and testing the envisioned trajectory in classroom situations for investigating *how* a trajectory works and can be improved, instead of trying to decide *whether* it works.

[2] This case is based upon: Doorman & Gravemeijer (2009).

The learning route – inspired by the domain history - is tried out and revised during teaching experiments in three tenth-grade classes. We collected data by video and audio taping whole class discussions and group work. The videotapes were used to analyse students' discourses and students' written materials with respect to the conjectured teaching and learning process.

Teaching experiment phase

We illustrate the change in how students think and talk about a model with the following episode. The trajectory starts with questions about a weather forecast. The teacher discusses the change of position of a hurricane with students: when will it reach land? This problem is posed as a leading question throughout the unit as a context for the need of grasping change. After the emergence of time series as useful tools for describing change of position, students work with situations that are described by stroboscopic photographs. The idea is that students come up with measurements of displacements, and that it makes sense to display them graphically for finding and extrapolating patterns. Two types of discrete graphs are discussed: graphs of displacements (distances between successive positions) and graphs of the total distance travelled. Note that discrete graphs are not introduced as an arbitrary symbol system, but emerge as models of discrete approximations of a motion, that link up with prior activities and students' experiences. By using the computer program Flash students are able to investigate many situations. During these activities their attention shifts from describing specific situations to properties of these discrete graphs and the relation with kinematical concepts.

Our findings confirm such a change in reasoning. In the beginning students refer to distances between successive positions. After a while they reason using the global shapes and properties of graphs and motion. An example of such reasoning concerns an exercise about a zebra that is running at constant speed and a cheetah that starts hunting the zebra. The question is whether the cheetah will catch up with the zebra. In the graphs the successive measurements of the zebra and the cheetah are displayed. The following discussion takes place between an observer and two students (Rob and Anna).

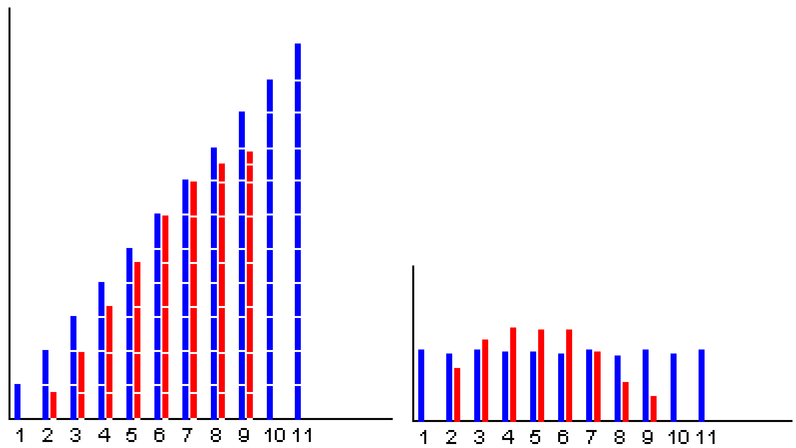


Figure 1. Distance-travelled and displacement graphs in Flash.

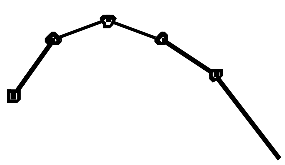
> Observer: Oh yes. So why did you choose the one for the total distance [left graph in Fig. 1]?
>
> Rob: Because it's the total distance that they cover and then you can-
>
> Anna: Then you can see if they catch up with each other.
>
> Observer: And can't you see that in the other [right graph]? There you can also see that the red [grey] catches up with blue [dark grey]?
>
> Rob: Yes, but -
>
> Anna: Yes, but that's at one moment. That only means that it's going faster at that moment but not that it'll catch up with the zebra.

Retrospective analysis

A difference between the displacements graph and the distance-travelled graph is the difference between the interpretations of the horizontal (time) axis. A value in the distance-travelled graph represents a distance from the start until the corresponding time, while a value in the displacements graph represents a distance in the corresponding time interval. Anna's last observation is an important step in the process of building the model of a velocity-time graph (and everything that comes with it).

The qualitative analyses show that during the practices students re-invent and develop graphical symbolisations, as well as the language and the scientific concepts that come with them. However, these inventions only became explicit after interventions by an observer or by the teacher. Additionally, we found that the teacher had a crucial role during the classroom discussions. It was not always easy to organise the discussions in line with the intended process. Sometimes the teacher reacted to students' contributions in terms of the inscriptions or concepts aimed at. In those cases students awaited further explanation. The discussions appeared to be especially productive when the teacher organised classroom discussions about students' contributions in such a way that the students themselves posed the problems that had to be solved, and reflected on their answers. In a second teaching experiment we arranged a setting where the teacher had more information about the possible contributions of the students and the way in which they could be organised. Additionally, we designed activities for classroom discussions. The HLT for the second teaching experiment is summarized in Figure 2. This summary shows the development of the tools that students used in their reasoning about change and how the transparency of the tools, the image that students have, is created by previous activities with preliminary representations. Furthermore, the figure illustrates the interaction between the development of these tools and the mathematical concepts and skills.

In this approach the construction and interpretation of graphs and the scientific concepts are rooted in the activities of the students through emerging models. This ensures that the mathematical and physical concepts aimed at are firmly rooted in students' understanding of everyday phenomena. On the basis of our findings we conclude that classroom discussions where students discuss their solutions and pose new problems to be solved, are essential for a learning process during which symbolisations and knowledge of motion co-evolve.

<table>
<tr><th>Tool</th><th>Imagery</th><th>Activity</th><th>Concepts</th></tr>
<tr><td rowspan="2">
time series (e.g. satellite photos & stroboscopic pictures)</td><td rowspan="2">real world representations signify real world situations</td><td>predicting motion (e.g. in the context of weather predictions)</td><td>displacements in equal time intervals as an aid for describing and predicting change</td></tr>
<tr><td colspan="2">should result in a feeling that the ability to predict motion with discrete data is an important issue</td></tr>
<tr><td rowspan="2">trace graphs of successive locations
</td><td rowspan="2">signifies a series of successive displacements in equal time intervals</td><td>compare, look for patterns in displacements and make predictions by extrapolating these patterns</td><td>displacements as a measure of speed, of changing positions, but difficult to extrapolate</td></tr>
<tr><td colspan="2">resulting in a willingness to find other ways to display displacements for viewing and extrapolating patterns in them</td></tr>
</table>

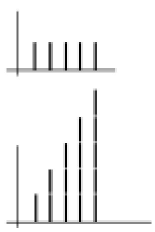
discrete 2-dim graphs 	signifies patterns in displacements of trace graphs (and cumulative)	compare patterns and use graphs for reasoning and making predictions about motion (also at certain moments: interpolate graphs) refine your measurements for a better prediction: displacements decrease	displacements depicting patterns in motion; linear line of summit in graph of displacements or graph of distances traveled; problems with predictions of instantaneous velocity
		should result in the need to know more about the relation between sums and differences, and in the need to know how to determine and depict velocity	

Figure 2: The HLT for the second teaching experiment

4. An exemplary case: Indonesian traditional games and the need for a standard measuring unit[3]

Background and design of HLT

In many countries, measurement is taught to young children as an isolated concept at a formal level. Teaching and learning linear measurement mostly focuses on the use of the ruler as an instrumental procedure. One possible effect of this approach is that students tend to perform a measurement as an instrumental procedure, without a consistent conceptual basis. The lack of a conceptual basis also plays a role in the inability of most students in grades 2 to 4 to correctly measure the length of an object that is not aligned with the first stripe of the ruler (Kamii & Clark, 1997; Kenney & Kouba in Van de Walle, 2005; and Lehrer et al, 2003).

In Indonesia, there are traditional games that, without any doubt, are related to measurement. Games like "gundu" (playing marble) and "benthik" embody linear measurement concepts including comparing, estimating and measuring distances. It is conjectured that the game playing provides a natural context for experience-based activities in which measurement concepts and skills can become meaningful and support further teaching and learning. Consequently, the main objective of this case was to contribute to a local instruction theory for the teaching and learning of linear measurement for Indonesian grade 2 students.. The students' situational reasoning within the game can be used as a source for the teaching and learning process to elicit the concepts of linear measurement. In addition, teachers can foster the learning of linear measurement by building on students' reasoning by referring to situations within the game and generalizing to other measuring practices and tools.

In the experience-based activities we used two Indonesian games, gundu and benthik. These Indonesian traditional games provide a natural situation for linear measurement in which students performed comparison and measurement of length in determining the winner of the games. The focus in the first Indonesian traditional game – gundu – is a comparison of lengths (the rules of the games can be found in the appendix). The winner of the game is the player whose marble is the nearest to a given circle, therefore students need comparison to determine the winner. Direct and indirect comparison activities are important, because they do not require dealing with numbers and units, and therefore direct students to focus on understanding length as the measurable attribute and the basic processes of measuring. We conjectured that students would use three kinds of benchmarks to compare the distances, namely: mental benchmark (i.e. mental estimation), body parts, and non-body parts such as pencil, stick, etc. In the second game – benthik –he winner is the team that obtains the

[3] This case is based upon: Wijaya, Doorman & Keijzer (2011).

bigger accumulative distances. This game changed focus from the concept of comparison of length to measurement of length.

A class discussion was always conducted after each game playing to elicit issues in measurement and to support and develop students' acquisition of the basic concepts of linear measurement. The purpose of the class discussion was not merely communicating some sensible idea or strategy, but also encouraging all students to share, discuss and develop their way of reasoning. Therefore, these class discussions also aimed to develop interactivity as an accepted norm in the classroom.

The experience-based measurement activities as the preliminaries of the instructional sequence aimed at providing a situational grounding in which the Indonesian traditional games provide and activate situational knowledge and strategies of linear measurement such as iterating hand spans and pencils to cover the distances. The class discussion after the game playing aimed to develop the situational level – in which students used their own hand spans to determine the winner – to the referential level when students started to consider a need to use a "common measurement unit" for fairness of the game. As the final activity, the formal measurement activities focused on the concept of a "standard measuring unit" and using a "standard measuring tool" (see Figure 3).

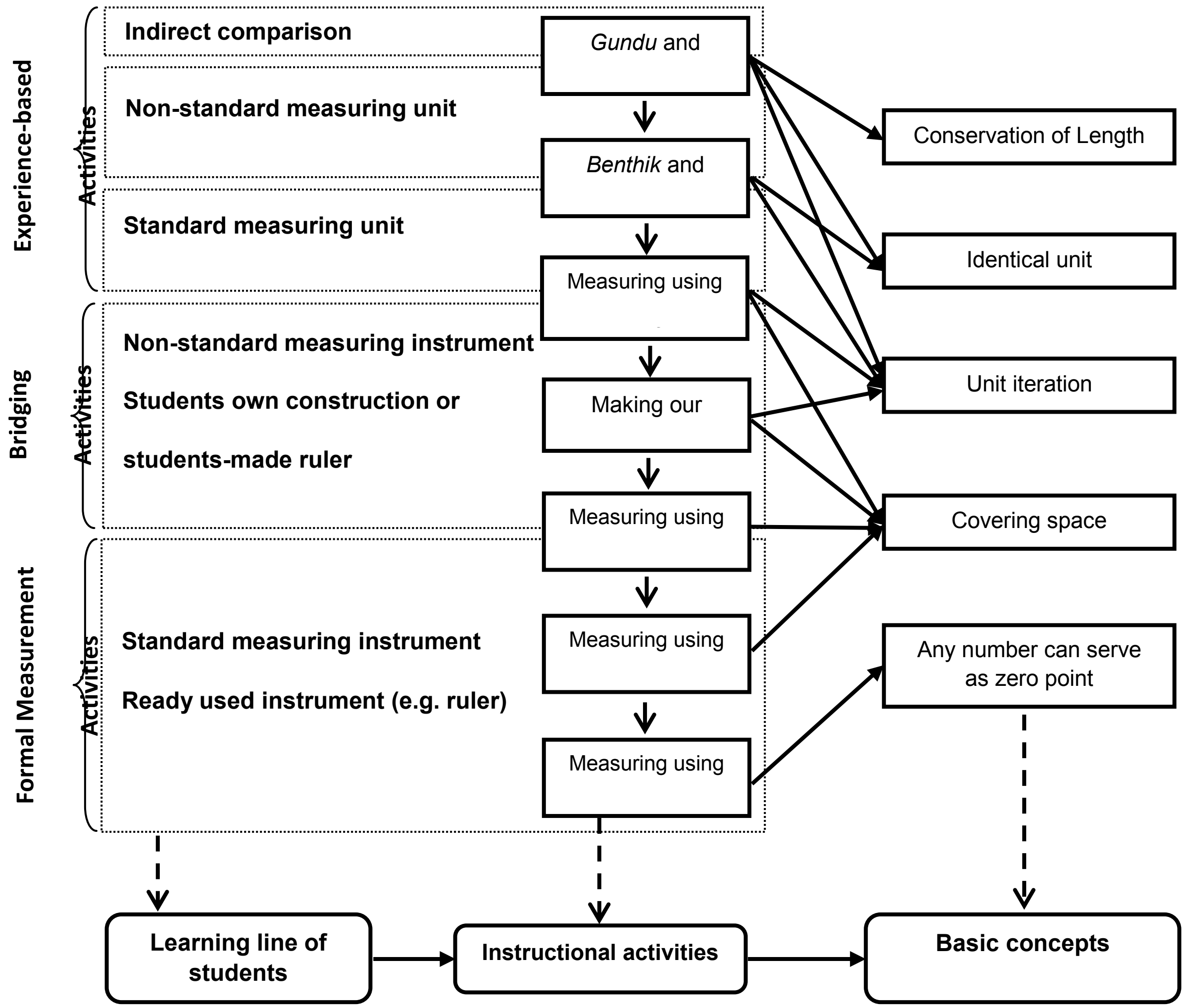


Figure 3. The main framework of experience-based activities for learning linear measurement

Teaching experiment phase

During the game playing, students started to use "third objects" as their measuring units and also started to realize that they needed to iterate the unit when the measured distances were longer than the measuring unit. At the beginning of playing gundu, students still used body parts (i.e. handspans and feet) and pencil to compare the distances. But in the last 15 minutes of playing, students started to think about other strategies when there was a conflict in which two marbles seemed to have the same distance to the circle, namely 3 spans in length. In fact, the distances of these marbles to the circle were different (i.e. about and spans) but both students adapted their last span to have a complete or integer span.

This fairness conflict stimulated students to consider a need for precision and, furthermore, to come up with an idea of a standard measuring unit. This 'need' or 'motive' for improving their measurement practice emerged during the playing. Finally, the students were able to determine the nearest marble when they used a piece of chalk to compare the distances from each marble to the circle. In this situation, the students started to consider that the choice of the unit size determines the precision of the measure.

The students' linear measurement experience from playing the game was combined with the teacher's story to elicit the basic concepts of linear measurement. The advantage of the fairness conflict in determining the winner within the situation of game playing was also shown in the class discussion. The fairness conflict could stimulate students to realize the need for a standard measuring unit because of the shared game playing experiences and intrinsic motivation to be fair. The following vignette illustrates how the fairness conflict plays this key role:

> Haya : The game is not fair if there are many students measuring the distances because the different size of steps will give different result (of measurement)
>
> Teacher : So … can we use different steps to measure the distances in our game?
>
> Students: No it is not because it is not fair
>
> Teacher : What should we do?

> Haya : In a game, we will have a fair game if there is only one person who measures the distances, because the different size of steps will give a different result (of measurement)

The game playing as situational level provided for an experience for students that supported them to move to the referential level. Haya proposed a solution to have a fair game by referring to a specific situation in the game playing. The phrase "…the different size of steps will give a different result" proposed by Haya illustrates that Haya became to understand the need for precision (i.e. the relation between the size of a measuring unit and the result of the measurement). Next, the teacher posed the problem about "different steps" to stimulate students to elicit the concept of a standard measuring unit. The question of the teacher and the word fair encourage Haya to come up with a standardization of the measuring unit, although she still used one person's body part as measuring unit. Haya also shows how she started to move from the referential level to the general level when she tried to generalize the solution that was indicated by the phrase "In a game…".

In the second traditional game, benthik, most students started to use a stick instead of handspans as the measuring unit. In this activity, iterating sticks became the model-of the activity of iterating hand spans. The students' understanding of a standard measuring unit was still developed in the class discussion following the game of benthik. In the class discussion, iterating sticks was also used to foster the idea of using a standard measuring unit. Therefore, the stick became the model-for reasoning about characteristics of measurement and measuring instruments.

Retrospective analysis

In summary we point on the importance of the fairness conflict during these teaching experiments. The fairness conflict seemed to contribute to stimulating students to conceive the idea of a standard measuring unit. However, student achievement at this stage was still at an informal knowledge. Consequently, the next important step in the instructional sequence was providing "bridging" activities to develop students' informal knowledge into more formal knowledge of linear measurement. The use of strings of beads is proposed to shift reasoning from iterating sticks to using a ruler.

This research aimed to contribute to formulating and developing a local instruction theory for teaching and learning of linear measurement in grade 2 of Indonesian elementary school. The local instruction theory with respect to the sequence of experience-based activities and the intended concept development for the teaching and learning of linear measurement is summarized in the local instruction theory (Figure 4). This summary shows the interaction between the development of the tools that were used and the acquisition of the mathematical concepts.

<table>
<tr><th>Activity</th><th>Tool</th><th>Imagery</th><th>Practice</th><th>Concept</th></tr>
<tr><td rowspan="2">Indonesian traditional games: playing gundu</td><td rowspan="2">Hand span, feet, marble</td><td rowspan="2"></td><td>Indirect comparison</td><td>Conservation of length
Emergence of a benchmark for indirect comparison</td></tr>
<tr><td colspan="2">The activity of iterating the benchmarks of comparison should become the focus for the introduction to measurement.</td></tr>
<tr><td rowspan="3">Indonesian traditional games: playing benthik</td><td rowspan="3">Hand span, feet, stick</td><td rowspan="3">Signifies that the "third object" in comparison become the measuring unit in measurement</td><td colspan="2">Playing benthik provides an opportunity to develop the use of "a third object" as a benchmark for indirect comparison, which becomes a measuring unit</td></tr>
<tr><td>Measuring as the development of indirect comparison</td><td>Identical unit and unit iteration</td></tr>
<tr><td colspan="2">The fairness conflict in the game could lead to the need for a standard measuring unit</td></tr>
</table>

Measuring using strings of beads	Strings of beads	Signifies the iteration of measuring unit, such as hand span, feet and marbles	Measuring and reasoning about activity of iterating and counting a measuring unit. A beginning of using standard measuring unit.	Standard measuring unit for the fairness and precision of measurement
			The use of strings of beads should shift the focus of learning process from measuring units to measuring instruments	

Figure 4. The final HLT of Indonesian games in teaching measurement

5. Conclusions

The two case studies both tried to create innovative approaches for topics in mathematics education. The focus on *symbolising* proved viable. The use of semiotic theories turned out useful for analysing the relationship between symbolising and development of meaning. We assume on the basis of these projects and prior research that a carefully designed trajectory of symbol and meaning development is necessary to support the learning of mathematics. In that process, students need to get opportunities for their own constructions and reflection on them. Realistic contexts proved important in that.

With respect to the methodology of design-based research, we consider the Hypothetical Learning Trajectory a useful instrument in all phases of design research. During the design phase it is the theoretically grounded vision of the learning process, which is specified for concrete instructional activities. During the teaching experiments, the HLT offers a framework for decisions during the teaching experiment and guides observations and data collections. In the retrospective analysis phase, the HLT serves as a guideline for data selection and offered conjectures that could be tested during the analysis. The final HLT is a reconstruction of a sequence of concepts, tools and instructional activities, which constitute the effective elements of a learning trajectory. In this manner, the result is a well-considered and empirically grounded local instruction theory. The HLT, together with a description of the cyclic process of design and research, enables others to retrace the learning process of the research team. Understanding the how and why of the specified steps makes it possible to let that learning process become your own and to adapt findings to your own context.

With these two examples we illustrated design-based research as a systematic approach for innovation-oriented studies. The close connections between design and theory development offer teachers and researchers opportunities to translate the results to their own teaching or researching practice.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245-274.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences, 2*, 141-178.

Cobb, P., Wood, T. & Yackel, E. (1992). A constructivist Alternative to the Representational View of Mind in Mathematics Education. Journal for Research in Mathematics Education, 23. 2-33.

Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., Gravemeijer, K. P. E., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and Symbolizing: The Emergence of Chains of Signification in One First-Grade Classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition theory: Social, semiotic, and neurological perspectives* (pp. 151-233). Hillsdale, NJ: Erlbaum.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*(1), 2-33.

de Lange, J., Burrill, G., Romberg, T., & van Reeuwijk, M. (1993). *Learning and Testing Mathematics in Context: The Case: Data Visualization*. Madison, WI: University of Wisconsin, National Center for Research in Mathematical Sciences Education.

Doorman, L.M. & Gravemeijer, K.P.E. (2009). Emergent modeling: discrete graphs to support the understanding of change and velocity. *ZDM: The International Journal on Mathematics Education, 41* (1), 199-211.

Edelson, D. C. (2002). Design Research: What We Learn When We Engage in Design. *Journal of the Learning Sciences, 11*(1), 105-121.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Reidel.

Glaser, B. G., & Strauss, A. L. (1967). *The Discovery of Grounded Theory; Strategies for Qualitative Research*. Chicago: Aldine Publishing Company.

Gravemeijer, K. P. E. (1994). Educational Development and Developmental Research. *Journal for Research in Mathematics Education, 25*, 443-471.

Gravemeijer, K. P. E. (1998). Developmental Research as a Research Method. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics Education as a Research Domain: A Search for Identity (An ICMI Study)* (Vol. 2, pp. 277-295). Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K.P.E. en Doorman, L.M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, vol 39(1-3)*, pp. 111-129.

Gravemeijer, K. P. E., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, Modeling, and Instructional Design. In P. Cobb & E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 225 - 273). Mahwah, NJ: Lawrence Erlbaum Associates.

Latour, B. (1990). Drawing Things Together. In M. Lynch & S. Woolgar (Eds.), *Representation in Scientific Practice* (pp. 19-68). Cambridge, MA: MIT Press.

Meira, L. (1995). Microevolution of mathematical representations in children's activity. *Cognition and Instruction, 13*(2), 269-313.

Roth, W.-M., & McGinn, M. K. (1998). Inscriptions: Toward a Theory of Representing as Social Practice. *Review of Educational Research, 68*(1), 35-59.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Steffe, L. P., & Thompson, P. W. (2000). Teaching Experiments Methodology: Underlying Principles and Essential Elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education*. Hillsdale, NJ: Erlbaum.

Strauss, A., & Corbin, J. (1988). *Basics of Qualitative Research. Techniques d Procedures for Developing Grounded Theory*. Thousand Oaks, CA: SAGE Publications.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics, 22*, 125-147.

Treffers, A. (1987). *Three Dimensions. A Model of Goal and Theory Description in Mathematics Instruction - The Wiskobas Project*. Dordrecht, the Netherlands: Reidel Publishing Company.

van Oers, B. (2000). The Appropriation of Mathematics Symbols: A Psychosemiotic Approach to Mathematics Learning. In P. Cobb & E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms; perspectivies on discourse, tools, and instructional design* (pp. 133-176). Mahwah, NJ: Lawrence Erlbaum Associates.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in Realistic Mathematics Education. *Educational Studies in Mathematics 54*(1), 9-35.

Walkerdine, V. (1988). *The Mastery of Reason*. London: Routledge.

Whitson, J. A. (1997). Cognition as a Semiotic Process: From Situated Mediation to Critical Reflective Transcendence. In D. Kirshner & J. A. Whitson (Eds.), *Situated Cognition Theory: Social, Semiotic, and Neurological Perspectives* (pp. 97-150). Hillsdale, NJ: Erlbaum.

Wijaya, A., Doorman, L.M. & Keijzer, R. (2011). Emergent Modelling: From Traditional Indonesian Games to a Standard Unit of Measurement. *Journal of Science and Mathematics Education in Southeast Asia, 34* (2), 149-173.